1.

Difficulty : Medium

Topics :

Elasticity,

$$Stress = \frac{F}{A} = \frac{Mg}{A}$$
$$= \frac{\rho(Volume).g}{A}$$
$$= \frac{\rho(9^3x^3)g}{9^2x^2}$$
$$= 9\,times$$

2.

Difficulty : Easy

Topics :

Kinematics,

Velocity becomes negative after same time and the negative slope is constant.

3.

Difficulty : Easy

Topics :

Work ,Energy and power,

$$f = -kv^2$$
$$10^{-2}a = -kv^2$$
$$a = -100kv^2$$
$$\frac{dv}{dt} = -100kv^2$$
$$\int_{v_0}^{\frac{v_0}{2}} = -100k\int_0^{10} dt$$
$$\left(-\frac{1}{v}\right)_{v_0}^{\frac{v_0}{2}} = -100k(t)$$
$$\frac{2}{v_0} - \frac{1}{v_0} = 100k.t$$
$$\frac{1}{v_0} = 100kt$$
$$k = \frac{1}{100v.t} = \frac{1}{100 \times 10 \times 10} = 10^{-4}kg\,m^{-1}$$

4.

Difficulty : Easy

Topics :

Work ,Energy and power,

$F = 6t$
or $a = \frac{F}{m} = \frac{6t}{1} = 6t$
or $\frac{dV}{dt} = 6t$
or $\int_0^V dV = \int_0^t 6t\,dt$
or $V = 3t^2$
At $t = 1sec,\ V = 3\,ms^{-1}$
$\therefore W = \Delta k = k_f - k_i$
$= \frac{1}{2}mv^2 - 0 = \frac{1}{2} \times 1 \times 9 = 4.5J.$

5.

Difficulty : Easy

Topics :

Rotational Mechanics,

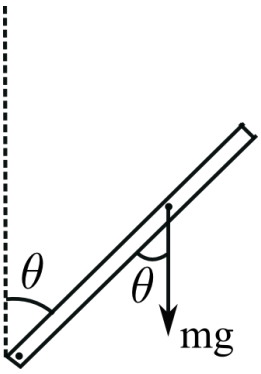
$m = \rho(\pi R^2 l)$
$dm = \rho\pi(2R.dRl + R^2 dl)$
$O = \rho\pi(2RldR + R^2 dl)$
$2Rl\,dR = -R^2 dl$
$I = \frac{MR^2}{4} + \frac{Ml^2}{12}$
$I = \frac{M}{12}(3R^2 + l^2)$
$dI = \frac{M}{12}(6R(dR) + 2l\,dl) = 0$
$6R\left(\frac{-R^2 dl}{2Rl}\right) + 2l\,dl = 0$
$\left(\frac{-3R^2}{l} + 2l\right) dl = 0$
$-3R^2 + 2l^2 = 0$
$2l^2 = 3R^2$
$\frac{l}{R} = \sqrt{\frac{3}{2}}$

6.

Difficulty : Medium

Topics :

Rotational Mechanics,



Torque of gravity about $O = mg \times \frac{L}{2}\sin\theta = I\alpha$

or $\alpha = \frac{mg\frac{L}{2}\sin\theta}{\frac{mL^2}{3}} = \frac{3g}{2L}\sin\theta$

7.

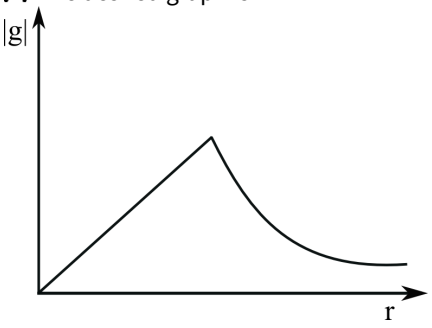
Difficulty : Easy

Topics :

Gravitation,

$$|g| = \begin{cases} \frac{GMr}{R^3}, & 0 \le r < R \\ \frac{GM}{r^2}, & r \ge R \end{cases}$$

$\therefore$ The desired graph is



8.

Difficulty : Easy

Topics :

Caloriemetry,

Heat lost by ball = Heat gain by calmimetre and water

$$100(0.1)(T-75) = 100(0.1)(75-30) + 170(1)(75-30)$$
$$10(T-75) = 10(45) + 170(45) = 180(45)$$
$$10(T-75) = 8100$$
$$T = 810 + 75 = 885^{o}C$$

9.

Difficulty : Easy

Topics :

Elasticity,

$$P = B\left|\frac{-\Delta V}{V}\right|$$

and $\Delta V = Vr\Delta T = 3V\alpha\Delta T$

$$\therefore P = \frac{K \times 3V\alpha\Delta T}{V}$$

or $\Delta T = \dfrac{P}{3K\alpha}$

10.

Difficulty : Easy

Topics :

Physics,
Heat and thermodynamics,

$$C_p - C_v = \frac{R}{M}$$

for hydrogen $C_p - C_v = \dfrac{R}{2} = a$

for nitrogen $C_p - C_v = \dfrac{R}{28} = b$

$$\frac{a}{b} = 14$$
$$a = 14b$$

11.

Difficulty : Easy

Topics :

Physics,
Heat and thermodynamics,

$$n_f - n_i = \frac{PV}{R}\left[\frac{1}{T_f} - \frac{1}{T_i}\right]$$
$$\simeq -2.5 \times 10^{25}$$

12.

Difficulty : Easy

Topics :

Simple harmonic motion,

The period of K.E graph is half of the period of displacement graph.
Comment :-
The points on the graph on x-axis should be differentiable.
In all option mentioned, the graph has cusps at the points on x-axis.

13.

Difficulty : Medium

Topics :

Wave optics,

$$\frac{c}{f'} = \frac{c}{f}\sqrt{\frac{1-\beta}{1+\beta}}\left[\beta = \frac{v}{c}\right]$$
$$\left[\beta = \frac{1}{2}\right]$$
$$\frac{1}{f'} = \frac{1}{f}\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{1}{3}}$$
$$f' = \sqrt{3}f$$
$$f' = 1.73(10)$$
$$f' = 17.3\ GHz$$

14.

Difficulty : Medium

Topics :

Electrostatics,

Since $\vec{\tau}_1 = \tau\hat{k}$, dipole moment is in xy plane

Consider $\vec{P} = P_x\hat{i} + P_y\hat{j}$

$\vec{\tau}_1 = \vec{P} \times \vec{E}_1$

$= (P_x\hat{i} + P_y\hat{j}) \times E\hat{i}$

$= P_yE = \tau$

$\vec{\tau}_2 = \vec{P} \times \vec{E}_2 = (P_x\hat{i} + P_y\hat{j}) \times \sqrt{3}E\hat{j}$

$= \sqrt{3}P_xE\hat{k}$

According to the problem $\vec{\tau}_2 = -\vec{\tau}_1$

$\Rightarrow \tau_2 = \tau_1 \Rightarrow \sqrt{3}P_xE = P_yE$

$\Rightarrow \dfrac{P_y}{P_x} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3}$

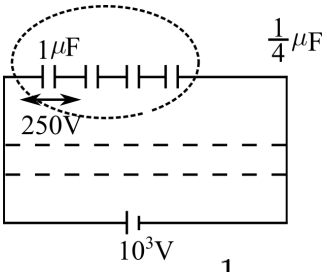
$\Rightarrow \theta = 60^o$

15.

Difficulty : Easy

Topics :

Capacitors,



8 rows, then Get $= \dfrac{1}{4} \times 8 = 2\mu F$

So minimum of capacitors are

$8 \times 4 = 32$

16.

Difficulty : Easy

Topics :

Current electricity,

$$i = \frac{E}{r + r_2}$$

By KVL : -

$$\Delta V + ir - E = 0$$

$$\Delta V = E - \frac{E}{r + r_2}.r$$

$$= \frac{Er_2}{r + r_2}$$

So charge on capacitor is $Q = C\Delta V$

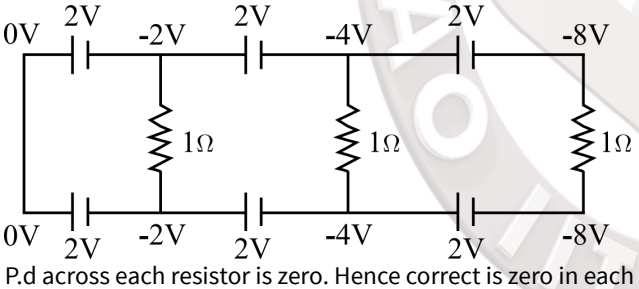
$$Q = \frac{CEr_2}{r + r_2}$$

17.

Difficulty : Easy

Topics :

Current electricity,



P.d across each resistor is zero. Hence correct is zero in each resistor.

18.

Difficulty : Easy

Topics :

Physics,

Topic : Classical Magnetism

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Time period $= T \times 10 = 2\pi\sqrt{\frac{I}{MB_H}}$

$$= 2 \times 3.14 \times \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} \times 10$$

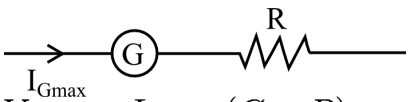
$$\simeq 6.65\ sec$$

19.

Difficulty : Easy

Topics :

Current electricity,



$V_{max} = I_{Gmax}(G + R)$
$10 = 5 \times 10^{-3}(15 + R)$
or $\frac{10}{5 \times 10^{-3}} = 15 + R$
or $2000 = 15 + R$
or $R = 1985$
or $R = 1.985 \times 10^{3}\Omega$

20.

Difficulty : Easy

Topics :

Electromagnetic Induction(EMI),

$\frac{d\phi}{dt} = iR$
or $d\phi = idt \times R$
or $\int_{\phi i}^{\phi f} d\phi = R \int idt$
or $\phi_f - \phi_i = R \times \int idt$
or $\phi_f - \phi_i = 100 \times \frac{1}{2} \times 0.5 \times 10$
or $\Delta\phi = 250Wb$

21.

Difficulty : Easy

Topics :

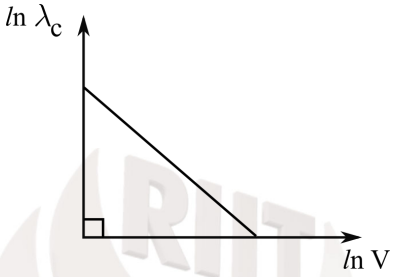
Modern Physics,

$$\lambda_c = \frac{hc}{eV}$$

$$\therefore ln\,\lambda_c = ln\left(\frac{hc}{eV}\right)$$

or $ln\,\lambda_c = ln\left(\frac{hc}{e}\right) - ln\,V$

$\therefore$ The graph of $ln\,\lambda_c$ Vs $\lambda_n V$ is as follows

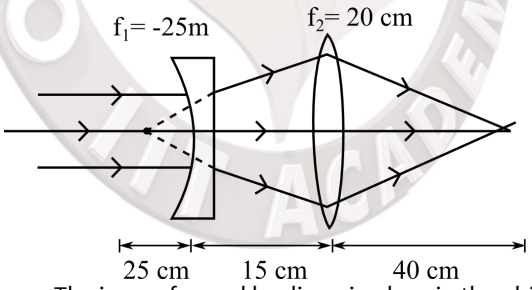


22.

Difficulty : Easy

Topics :

Ray Optics,



Diverging lens forms the image at its focus. The image formed by diverging lens in the object to converging lens.

$\therefore$ Distance of the object from converging lens is $40\ cm$ i.e., object is at double focal point.

$\therefore$ Image forms at double focal point on the other side of the lens.

23.

Difficulty : Easy

Topics :

Wave optics,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$\Rightarrow n_1\lambda_1 = n_2\lambda_2 \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650}$$

$$\frac{n_1}{n_2} = \frac{4}{5}$$

4th Bright of λ_1 will coincide with 5th bright of λ_2

$$\lambda_{\min} = \frac{4\lambda_1 D}{d}$$

$$= \frac{45(650\times10^{-9})(1.5)}{0.5\times10^{-3}}$$

$$= \frac{6(650)\times10^{-8}}{5\times10^{-4}}$$

$$= 6(13)\times10^{-4}$$

$$= 78\times10^{-4}m$$

$$= 7.8\,mm$$

24.

Difficulty : Easy

Topics :

Modern Physics,

$$(m) \longrightarrow \underset{\text{rest}}{V\left(\frac{m}{2}\right)}$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = \frac{m - \frac{m}{2}}{m + \frac{m}{2}} \times v = \frac{v}{3}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2\times mv}{m + \frac{m}{2}} = \frac{4v}{3}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2v_2}{m - 1v_1} = \frac{\frac{m}{2}}{m} \times \frac{\frac{4v}{3}}{\frac{v}{3}} = 2:1$$

25.

Difficulty : Easy

Topics :

Modern Physics,

$$\frac{4E}{3} - E = \frac{hc}{\lambda_2} \Rightarrow \frac{E}{3} = \frac{hc}{\lambda_2}$$
$$2E - E = \frac{hc}{\lambda_1} \Rightarrow E = \frac{hc}{\lambda_1}$$
$$\frac{1}{3} = \frac{\lambda_1}{\lambda_2} = r$$

26.

Difficulty : Easy

Topics :

Modern Physics,

$N = N_o\, e^{-\lambda t} \rightarrow$ no. of undecayed nucleus no. of decyaed nucleus is $= N_o - N$

$$\frac{N_o - N}{N} = 0.2N_o = N_2\, 0.3N$$
$$N_o = 1.3N = 1.3(N_o e^{-\lambda t})$$
$$e^{-\lambda t} = \frac{1}{1.3}$$
$$\lambda t = ln(1.3)$$
$$\frac{ln2}{T} t = ln(1.3)$$
$$t = T\frac{ln(1.3)}{ln2}$$

27.

Difficulty : Easy

Topics :

Physics,

Topic - Semiconductor
Voltage gain of the amplifier is

$$A_v = \frac{v_0}{v_i} = \frac{\Delta V_{CE}}{r \Delta I_B}$$
$$= \frac{-B_{ac} R_L}{K}$$

-Ve sign indicates phase difference is 180°

28.

Difficulty : Easy

Topics :

Physics,

Topic - Communication System

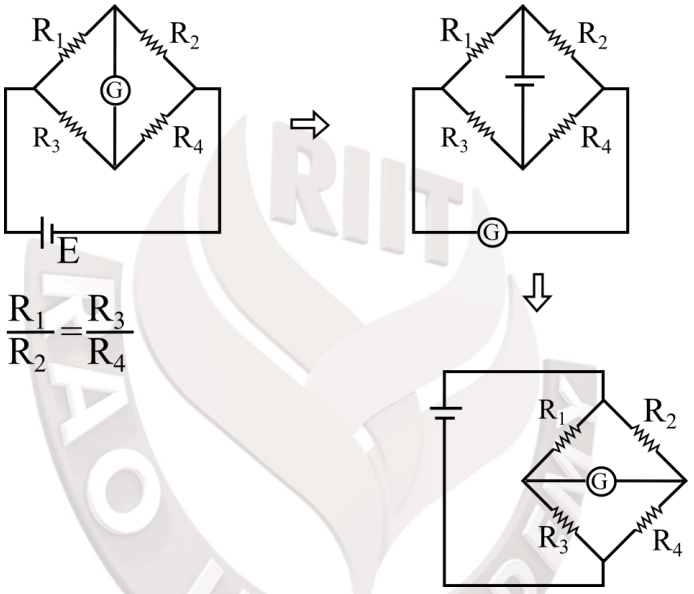
ω_m will not be there in the frequencies.

29.

Difficulty : Medium

Topics :

Current electricity,



$$\frac{R_1}{R_2}=\frac{R_3}{R_4}$$

Null point does not change

30.

Difficulty : Easy

Topics :

Surface tension,

$$\frac{\Delta T}{T}\times 100=\frac{\Delta D}{D}\times 100+\frac{\Delta h}{h}\times 1000+\frac{\Delta g}{g}\times 100$$
$$=\frac{0.01}{1.25}\times 100+\frac{0.01}{1.45}\times 100+\frac{0.01}{9.8}\times 100$$
$$=\frac{1}{1.25}+\frac{1}{1.45}+\frac{1}{9.8}$$
$$=0.8+0.689+0.102$$
$$=1.59\approx 1.5\%$$

31.

Difficulty : Easy

Topics :

Chemistry,

Topic : Thermochemistry

$C_{(graphite)} + O_{2(g)} \rightarrow CO_2(g)\ \Delta_r H^o = -393.5\ kJ/mole$ .....(1)

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(\ell) \quad \Delta H_r^o = -285.8\ kJ/mole$ ....(2)

$CO_2(g) + 2H_2O(\ell) \rightarrow CH_4(g) + 2O_2(g) \quad \Delta H_r^o = +890.3\ kJ/mole$ .....(3)

(1)+2 × (2) and +(3) $= -393.5 + 2(-285.8) + 890.3$

$= -74.8\ kJ/mole$

32.

Difficulty : Medium

Topics :

Chemistry,
Mole concept,

$M_2CO_3 + 2HCl \longrightarrow 2MCl + H_2O + CO_2$

$1\ gram \qquad 0.01186\ mole$

$x\ gram \qquad 1\ Mole$

$\Rightarrow x = \frac{1}{0.01186} = 84.3\ g$ Molar mass

33.

Difficulty : Easy

Topics :

Chemistry,
Thermodynamics,

From First law of Thermodynamics

$\Delta U = q + w$

$\therefore$ for an Adiabatic process

$q = 0$

$\therefore \Delta U = W(\text{Adiabatic work})$

34.

Difficulty : Easy

Topics :

Chemistry,
Surface Chemistry,

Tyndall effect is observed when

(1) The diameter fo the dispersed particle is not much smaller than the wavelength of the light used.

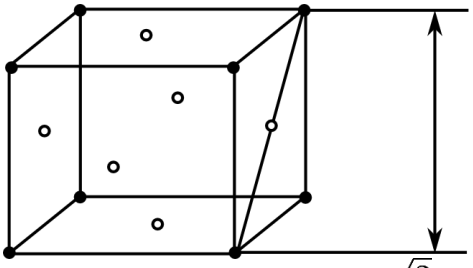
(2) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

35.

Difficulty : Medium

Topics :

Chemistry,
Solid State,



Length of face diagonal $= \sqrt{2}a$

$\therefore$ Distance of closest approach $= \dfrac{\sqrt{2}a}{2} = \dfrac{a}{\sqrt{2}}$

36.

Difficulty : Medium

Topics :

Chemistry,
Electrochemistry,

$E^o_{Cl_2/Cl^-} = 1.36\ V,\ E^o_{Cr^{+3}/Cr} = -0.74\ V,\ E^o_{Cr_2O_7^{2-}/Cr^{+3}} = 1.33\ V,$
$E^o_{MnO_4^-/Mn^{+2}} = 1.51\ V$

Strongest reducing agent is the atom/ ion which is easily oxidised i.e. having the largest value of standard oxidation potential (SOP)

$E^o_{Cr/Cr^{+3}} = +0.74\ V$ is the maximum value.

37.

Difficulty : Medium

Topics :

Chemistry,
Solutions,

$\Delta T_F = i \times K_F \times m$

$\therefore i = \dfrac{\Delta T_F}{K_F \times m}$

Since, $i_{\text{association}} = 1 - \left(1 - \dfrac{1}{n}\right)\alpha$

$= 1 - \left(1 - \dfrac{1}{2}\right)\alpha$

$= 1 - \alpha/2$

$\therefore \left(1 - \dfrac{\alpha}{2}\right) = \dfrac{0.45 \times 20 \times 60}{5.12 \times 0.2 \times 1000}$

$\therefore \left(1 - \dfrac{\alpha}{2}\right) = \dfrac{0.45 \times 1200}{200 \times 5.12} = \dfrac{2.70}{5.12}$

$\therefore \alpha = 0.943$ or $94.3\ \%$

38.

Difficulty : Easy

Topics :

Chemistry,
Atomic Structure,

$r_n = r_0 \times \frac{n^2}{Z}$; n = orbit number, $r_0 = 0.529\mathring{A} = \text{Radius of 1st orbit in H - atom}$

$r_2 = 0.529 \times 4 = 2.126\ \mathring{A}$

39.

Difficulty : Medium

Topics :

Chemistry,
Chemical Kinetics,

$K = A\ e^{-Ea/RT}$

$\frac{K_2}{K_1} = e^{(Ea_1 - Ea_2)/RT}$ (At 300K)

$In\left(\frac{K_2}{K_1}\right) = \frac{10 \times 10^3}{8.314 \times 300} \approx 4$

40.

Difficulty : Easy

Topics :

Chemistry,
Ionic equilibrium,

For the hydrolysis of a salt of weak acid and weak base

$pH = 7 + \frac{1}{2}\left[pK_a - pK_b\right]$

$pH = 7 + \frac{1}{2}\left[3.2 - 3.4\right] = 6.9$

41.

Difficulty : Difficult

Topics :

Chemistry,
S-block,

Mg form basic carbonate $\left[MgCO_3.Mg(OH)_2\right]$ while lithium from normal carbonate $\left(Li_2Co_3\right)$.

42.

Difficulty : Medium

Topics :

Chemistry,
Chemical Bonding,

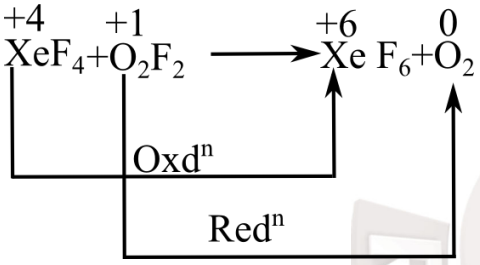
CO is diamagnetic only and has bond order 3.0. Rest all are paramagnetic $(O_2, B_2 \text{ and } NO)$

43.

Difficulty : Medium

Topics :

Chemistry,
P-block,



44.

Difficulty : Medium

Topics :

Chemistry,

Topic : Environmental science
$F^- =$ Above 2 ppm causes brown mottling of teeth
$SO_4^{-2} =$ Above 500 ppm causes laxative
$NO_3^- =$ Above 50 ppm causes blue baby syndrome.

45.

Difficulty : Easy

Topics :

Chemistry,

Topic : Periodic properties
$O^{2-}, F^-, Na^+, Mg^{+2} = Mg^{+2} = 10e^-$ in each species (all are isoelectronic).

46.

Difficulty : Medium

Topics :

Chemistry,
S-block,

$$NaOH + Cl_2 \longrightarrow NaCl + NaClO + H_2O$$

cold - dil

47.

Difficulty : Medium

Topics :

Chemistry,
d-block,

$$\underset{\text{Acidic}}{ZnO} + \underset{\text{Basic}}{Na_2O} \longrightarrow \underset{\text{SALT}}{Na_2ZnO_2}$$

$$\underset{\text{Basic}}{ZnO} + \underset{\text{Acidic}}{CO_2} \longrightarrow \underset{\text{SALT}}{ZnO_3}$$

48.

Difficulty : Difficult

Topics :

Chemistry,

Topic : Qualitative analysis

$$\underset{(X)}{Na_2C_2O_4} + \underset{\text{conc}}{H_2SO_4} \longrightarrow \text{Effervescence } (CO_2 + CO)$$

$$\xrightarrow{CaCl_2(aqu)} \text{White ppt } (CaC_2O_4)$$

$*$ '$\overset{+2}{C}O$' is responsible for decolourisation of acidic solution of $KMnO_4$.

49.

Difficulty : Medium

Topics :

Chemistry,
Mole concept,

H is 10% by mass
$\therefore$ H is 7.5 kgs
It replaced by ^2H weight is doubled i.e., 15 kg
$\therefore$ 7.5 kg increase

50.

Difficulty : Difficult

Topics :

Chemistry,

Topic : coordination

No of moles of ions $= \dfrac{1.2 \times 10^{22}}{6 \times 10^{23}}$

$= 0.02$ moles.

0.01 moles of $COCl_3.6H_2O$ gives 0.02 moles of Cl^- ions

$\therefore$ No. of $Cl^{\ominus}$ outside the co - ordination sphere should be 2

$[C_0(H_2O)_5Cl]Cl_2.H_2O.$

51.

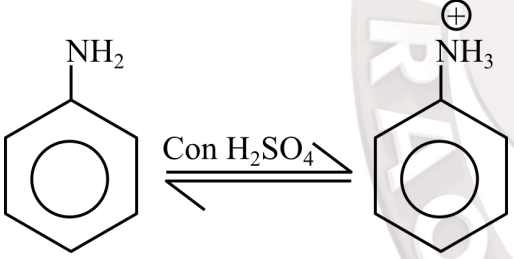
Difficulty : Medium

Topics :

Chemistry,

Topic - Nitrogen containing compounds

Aniline undergoes protination in con H_2SO_4, so it undergoes metanitration



52.

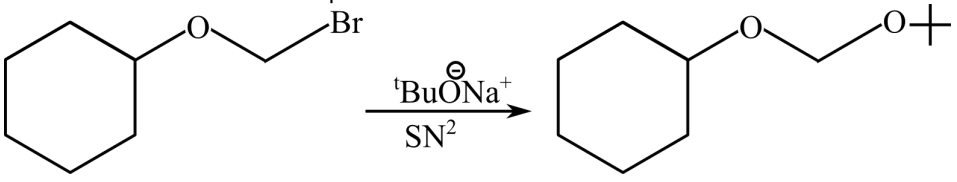
Difficulty : Medium

Topics :

Chemistry,

Topic - Alkylhalides

To shown decolourisation compound must be unsaturated



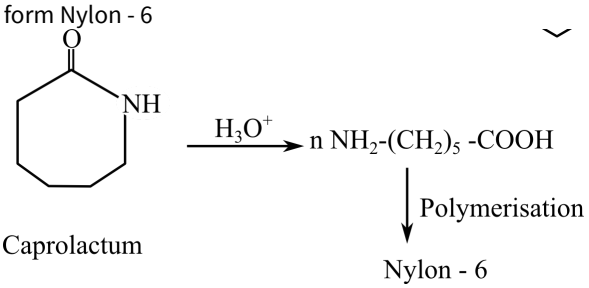
53.

Difficulty : Medium

Topics :

Chemistry,
Polymers,

Nylon - 6 is prepared from caprolactum,caprolactum on hydrolysis it gives aminocarboxylicacid it undergoes polymerisation to form Nylon - 6



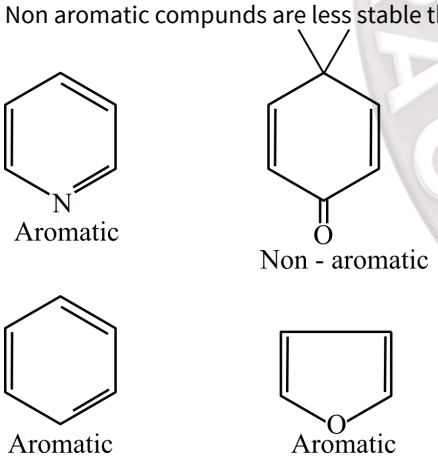
54.

Difficulty : Easy

Topics :

Chemistry,
GOC,

Non aromatic compunds are less stable than aromatic



55.

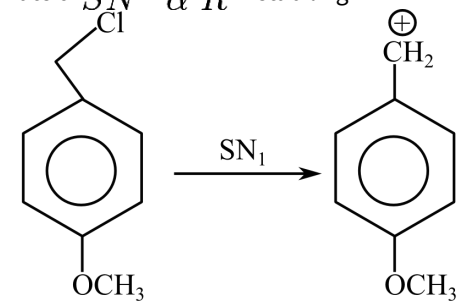
Difficulty : Easy

Topics :

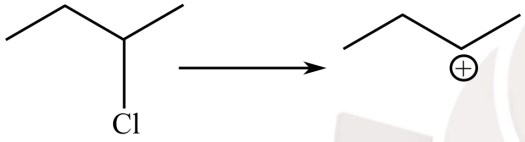
Chemistry,

Topic - Alkylhalides

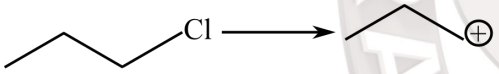
Rate of $SN^1 \ \alpha \ R^+$ stabiling



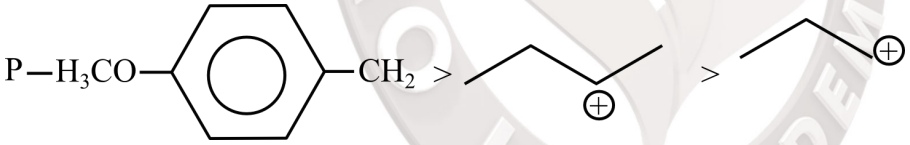
(III)



(I)



(II)



$\therefore$

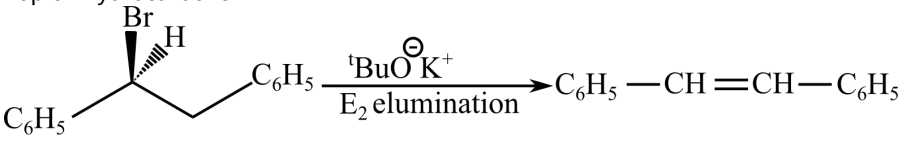
56.

Difficulty : Easy

Topics :

Chemistry,

Topic - Hydrocarbons



(+)

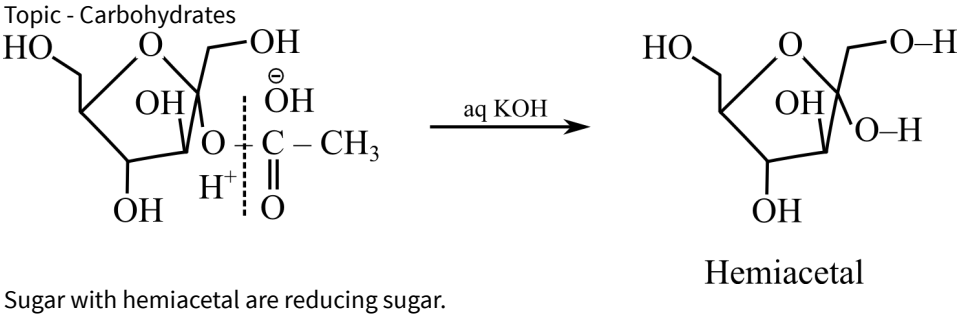
57.

Difficulty : Difficult

Topics :

Chemistry,

Topic - Carbohydrates



Hemiacetal

Sugar with hemiacetal are reducing sugar.

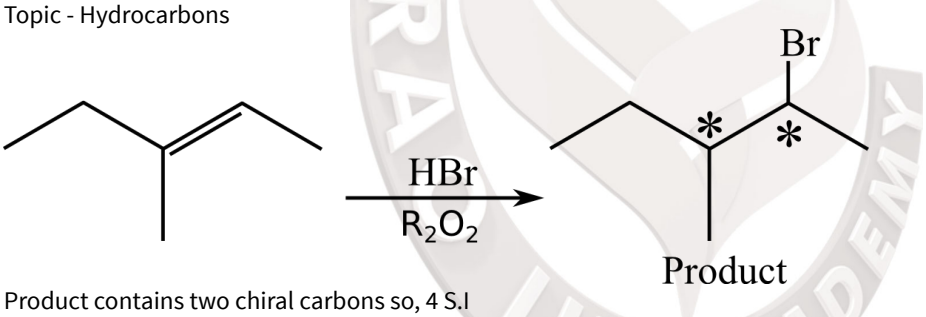
58.

Difficulty : Easy

Topics :

Chemistry,

Topic - Hydrocarbons



Product

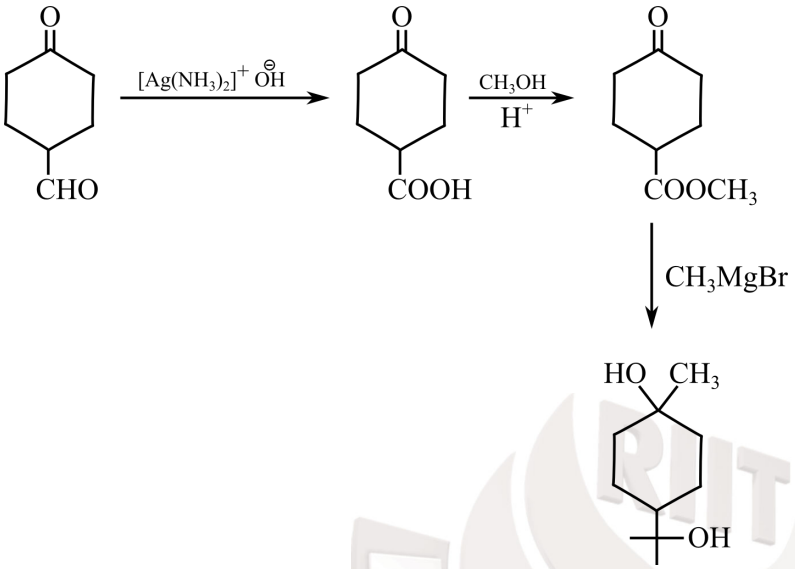
Product contains two chiral carbons so, 4 S.I

59.

Difficulty : Difficult

Topics :

Chemistry,
CARBONYLS,

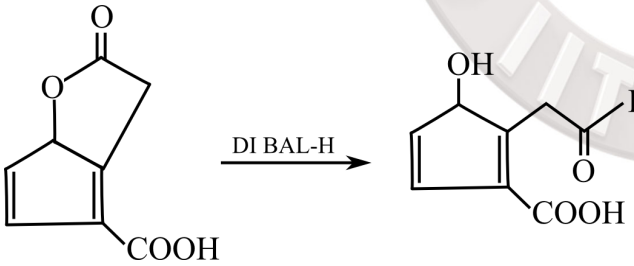


60.

Difficulty : Difficult

Topics :

Chemistry,
CARBONYLS,



DIBAL-H can reduce esters only, not carboxylicacids

61.

Difficulty : Medium

Topics :

Functions,
One-One Function (Injection),
Onto Function (Surjection),

$$f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}}{1+\frac{1}{x^2}}=\frac{x}{1+x^2}=f(x)$$

$\Rightarrow f\left(\frac{1}{2}\right)=f(2)\Rightarrow f(x)$ is many one

$y=\frac{x}{1+x^2}\Leftrightarrow yx^2-x+y=0$

$\Rightarrow \Delta\geq 0$

$\Leftrightarrow 1-4y^2\geq 0$

$\Leftrightarrow y\in\left[-\frac{1}{2},\frac{1}{2}\right]$

Range = Co-domain $\Rightarrow$ onto

62.

Difficulty : Medium

Topics :

Quadratic Equations,

$nx^2+n^2x+(0.1+1.2+2.3+....+(n-1)n)=10n$

$\Leftrightarrow nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$

as $n\neq 0$

$\Leftrightarrow 3x^2+3nx+n^2-31=0$

$|\alpha-\beta|=1\Leftrightarrow\frac{\sqrt{D}}{a}=1$

$\Leftrightarrow D=a^2\Leftrightarrow 9n^2-4.3(n^2-31)=9$

$\Leftrightarrow n^2=121\Leftrightarrow n=11$ as $n>0$

63.

Difficulty : Easy

Topics :

Complex Numbers,
Cube roots of unity,

$\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i, \Rightarrow \omega^3 = 1$ i.e. ω is non-real cube root of unity

and $\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3(\omega^2 - \omega)$$

$$\Rightarrow k = \omega^2 - \omega = -\sqrt{3}i = \boxed{-z}$$

64.

Difficulty : Easy

Topics :

Determinants & Matrices,
Adjoint of square Matrix,

$$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$\Rightarrow 3A^2 + 12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\Rightarrow adj(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

65.

Difficulty : Medium

Topics :

Determinants & Matrices,
Cramer's Rule,

As system no solution then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$

$\Rightarrow a^2 - 2a + 1 = 0 \Leftrightarrow a = 1$

when $a = 1$ then both equation $x + y + z = 1$ and $x + ay + z = 1$

obeserv that if $b = 1$ then no solutions

if $b \neq 1$ then infinite solutions

66.

Difficulty : Medium

Topics :

Permutations & Combinations,

	X	Y
M	3	4
W	4	3

	Case (i)		Case (ii)		Case (iii)		Case (iv)	
	X	Y	X	Y	X	Y	X	Y
M	3	0	2	1	1	2	0	3
W	0	3	1	2	2	1	3	0

$^3C_3.^4C_0.^4C_0.^3C_3 + {}^3C_2.^4C_1.^4C_1.^3C_2 + {}^3C_1.^4C_2.^4C_2.^3C_1 + {}^3C_0.^4C_3.^4C_3.^3C_0$

$= 485$

67.

Difficulty : Easy

Topics :

Binomial Theorem,

$$= \left({}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}\right) - \left({}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10}\right)$$
$$= \frac{1}{2}\left({}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{20}\right) - (2^{10} - 1)$$
$$= \frac{1}{2}(2^{21} - 2) - 2^{10} + 1$$
$$= 2^{20} - 2^{10}$$

68.

Difficulty : Medium

Topics :

Sequence & Series,

$$(15a)^2 + (3b)^2 + (5c)^2 - (45ab) - (15bc) - (75ac) = 0$$
$$\Leftrightarrow (15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2 = 0$$
$$\Leftrightarrow 15a = 3b = 5c$$
$$\Leftrightarrow a = k, b = 5k, c = 3k$$
$$\Rightarrow b, c, a \text{ are in AP}$$

69.

Difficulty : Medium

Topics :

Sequence & Series,
Functions,

$$\text{Put } y=0 \Rightarrow f(x)=f(x)+f(0)+0$$
$$\Rightarrow \ f(0)=0 \Rightarrow c=0$$
$$\text{Put } y=-x \Rightarrow f(0)=f(x)+f(-x)-x^2$$
$$\Leftrightarrow \ 0=ax^2+bx+ax^2-bx-x^2$$
$$\Rightarrow \ a=\frac{1}{2}$$
$$a+b+c=3 \Rightarrow b=\frac{5}{2}$$
$$\Rightarrow f(x)=\frac{x^2+5x}{2}$$
$$\Rightarrow \ \sum_1^{10} f(n)=\frac{1}{2}\sum_1^{10} n^2+\frac{5}{2}\sum_1^{10} n$$
$$=\frac{1}{2}\frac{10.11.21}{6}+\frac{5}{2}.\frac{10.11}{2}$$
$$=\frac{10.11}{2}\left(\frac{7}{2}+\frac{5}{2}\right)=330$$

70.

Difficulty : Easy

Topics :

Limits,

$$\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$$
$$=\frac{1}{8}\lim_{x\to\frac{\pi}{2}}\frac{(\cot x-\cos x)}{\left(\frac{\pi}{2}-x\right)^3}$$
$$=\frac{1}{8}\lim_{x\to\frac{\pi}{2}}\frac{\cos x.(1-\sin x)}{\sin x.\left(\frac{\pi}{2}-x\right)^3}$$
$$=\frac{1}{8}\lim_{x\to\frac{\pi}{2}}\frac{1}{\sin x}.\frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)}.\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)^2}$$
$$=\frac{1}{8}\cdot 1\cdot 1\cdot\frac{1}{2}=\frac{1}{16}$$

71.

Difficulty : Medium

Topics :

Math,
Differentiation of one function with respect to another function,

Put $9x^3 = t^2$ and $t \geq 0$

$$y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right) = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

as $x \in \left(0, \frac{1}{4}\right) \Rightarrow t \in \left(0, \frac{3}{8}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{2t}{1-t^2}\right) = 2\tan^{-1} t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{2}{1+t^2} \cdot 3.\frac{3}{2}x^{1/2}$$

$$= \frac{9\sqrt{x}}{1+9x^3} = \sqrt{x}\, g(x)$$

$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

72.

Difficulty : Medium

Topics :

Application of derivatives,

$$\log y = \log(x+6) - \log(x-2) - \log(x-3)$$

$$\Rightarrow \frac{y'}{y} = \frac{1}{x+6} - \frac{1}{x-2} - \frac{1}{x-3}$$

$$\Rightarrow \frac{y'(0)}{y(0)} = \frac{1}{6} + \frac{1}{2} + \frac{1}{3} = 1$$

$$\Rightarrow y'(0) = y(0) = 1 \text{ (Slope of tangent)}$$

$$\Rightarrow \text{Slope of normal} = -1$$

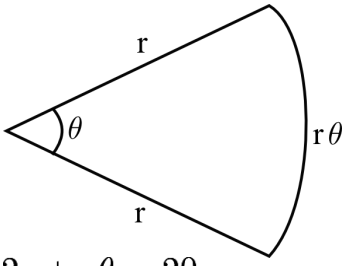
$\Rightarrow$ Equation of normal at $(0, 1)$ will be $x + y = 1$ which is satisfied by $\left(\frac{1}{2}, \frac{1}{2}\right)$ only in the given options.

73.

Difficulty : Easy

Topics :

Application of derivatives,



$2r + r\theta = 20$

$\Rightarrow \quad \text{Area of sector} = \frac{1}{2}r^2\theta = 10r - r^2$

$= 25 - (r-5)^2 \leq 25$

$\Rightarrow \quad \text{Maximum area} = 25$

74.

Difficulty : Easy

Topics :

Math,
Indefinite integration,

$$I_{n+2} + I_n = \int \left(\tan^{n+2} x + \tan^n x\right)\, dx$$

$$= \int \tan^n x(\tan^2 x + 1)\, dx$$

$$= \int \tan^n x \sec^2 x\, dx$$

$$= \frac{\tan^{n+1} x}{n+1} + c$$

$\Rightarrow \quad I_4 + I_6 = \frac{\tan^5 x}{5} + C$ (Put n = 4)

$\Rightarrow \quad \boxed{a = \frac{1}{5}, b = 0}$

75.

Difficulty : Easy

Topics :

Math,
Definite integration,

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{\sin^2 x} dx$$

$$\int_{\pi/4}^{3\pi/4} (\text{cosec}^2 x - \text{cosec}x \cot x) dx$$

$$= \text{cosec}x - \cot x]_{\pi/4}^{3\pi/4}$$

$$= (\sqrt{2}+1) - (\sqrt{2}-1) = 2$$

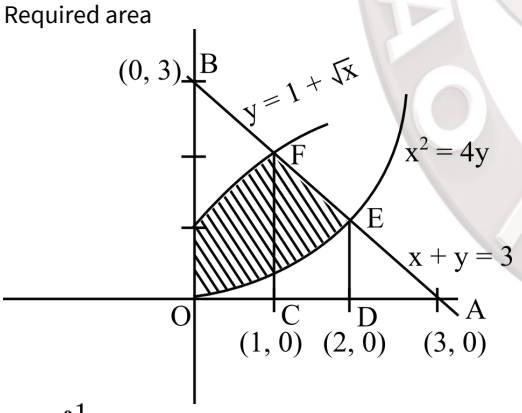
76.

Difficulty : Difficult

Topics :

Math,
Area under curve,

Required area



$$= \int_0^1 (1+\sqrt{x}) dx + \text{Area of trapizium } FCDE - \int_0^2 \frac{x^2}{4} dx$$

$$= \left[x + \tfrac{2}{3}x^{3/2}\right]_0^1 + \frac{1}{2}[2+1] \times 1 - \frac{1}{12}[x^3]_0^2$$

$$= 1 + \frac{2}{3} + \frac{3}{2} - \frac{2}{3} = \frac{5}{2} \text{ sq. units}$$

77.

Difficulty : Easy

Topics :

Differential Equations,

$$(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{\cos x}{2+\sin x}y=\frac{-\cos x}{2+\sin x} \qquad \text{...(I)}$$

I.F. of equation (I) is $e^{\int \frac{\cos x}{2+\sin x}dx}=2+\sin x$

Hence answer is

$$y(2+\sin x)=\int \frac{-\cos x}{2+\sin x}\times(2+\sin x)dx=-\sin x+c$$

As $y(0)=1 \quad \Rightarrow \quad c=2$

So $y(2+\sin x)=2-\sin x$

Now $y\left(\frac{\pi}{2}\right)=\frac{1}{3}$

78.

Difficulty : Easy

Topics :

Straight lines,

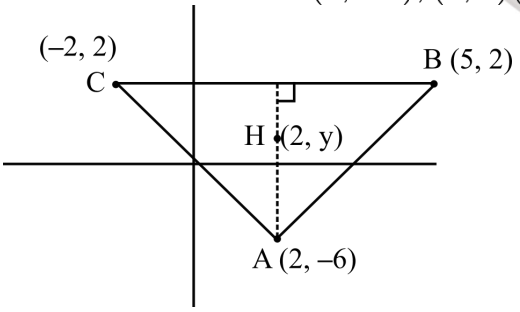
$$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}=\pm 56$$

$\Rightarrow \; 5k^2+13k+66=0$ or

$5k^2+13k-46=0$

as $k\in I \quad \Rightarrow \quad k=2$

$\Rightarrow$ Triangle has co-ordinates $(2,-6),(5,2)(-2,2)$



As one side is horizontal $\Rightarrow$ one altitude will be vertical $\Rightarrow$ arthocentre $=(2,y)$

$$\text{Slope of } BH\cdot \text{Slope of } AC=-1$$

$$\Rightarrow \quad \frac{y-2}{2-5}\cdot\frac{2+6}{-2-2}=-1 \Rightarrow \; y=\frac{1}{2}$$

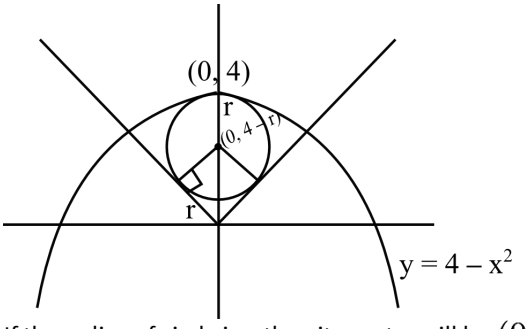
$$\Rightarrow \quad \text{orthocentre}=\left(2,\frac{1}{2}\right)$$

79.

Difficulty : Medium

Topics :

Circles,
Conic Sections,



If the radius of circle is r, then its centre will be $(0, 4-r)$

In an isoscales right angled Δ above, $r^2 + r^2 = (4-r)^2 \quad \Rightarrow \quad r = 4(\sqrt{2}-1)$

80.

Difficulty : Easy

Topics :

Conic Sections,

$e = \frac{1}{2}$ and $\frac{a}{e} = 4 \Rightarrow a = 2$

Again $b^2 = a^2(1-e^2) = 3$

Hence equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

and equation of normal at point (x_1, y_1) is:

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

Equation of normal at point $\left(1, \frac{3}{2}\right)$

is $4x - \frac{3y}{3/2} = 1 \quad \Rightarrow \boxed{4x - 2y = 1}$

81.

Difficulty : Medium

Topics :

Math,
Conic Sections,

$ae = 2 \Rightarrow a^2 + b^2 = a^2e^2 = 4$

$\Rightarrow$ Hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{4-a^2} = 1$

it is satisfied by $\left(\sqrt{2}, \sqrt{3}\right)$

$\Rightarrow \quad \frac{2}{a^2} - \frac{3}{4-a^2} = 1 \Rightarrow a = 1$

$\Rightarrow$ Hyperbola is $x^2 - \frac{y^2}{3} = 1$

$\Rightarrow$ equation of tangent at $\left(\sqrt{2}, \sqrt{3}\right)$ will be

$\sqrt{2}x - \frac{\sqrt{3}y}{3} = 1 \quad i.e. \quad \sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

Which is satisfied by $\boxed{(2\sqrt{2}, 3\sqrt{3})}$

82.

Difficulty : Easy

Topics :

Math,
Three Dimensional Geometry,

Normal perpendicular to both the lines

$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$

$\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$

Normal vector parallel to perpendicular $(\vec{n}_1 \times \vec{n}_2)$

where $\vec{n}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$

& $\vec{n}_2 = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{k} + 3\hat{k}$

Hence equation of plane is $5(x-1) + 7(y+1) + 3(z+1) = 0$

$\Rightarrow \quad 5x + 7y + 3z + 95 = 0 \quad ....(1)$

Now distance from point (1, 3, -7) to the plane is $\left|\frac{5 + 21 - 21 + 5}{\sqrt{83}}\right| = \frac{10}{\sqrt{83}}$ units

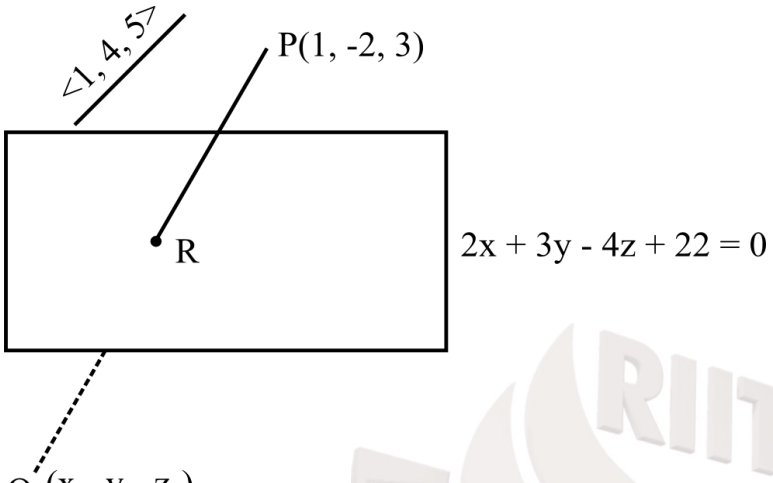
83.

Difficulty : Medium

Topics :

Math,
Three Dimensional Geometry,

Line PQ parallel to $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$



$\Rightarrow \quad \frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$

Point R is $(\lambda+1, 4\lambda-2, 5\lambda+3)$

As point R lies on plane then

$2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$

$\lambda = 1$

Hence point R is (2, 2, 8)

$\Rightarrow \quad PR = \sqrt{1+16+25} = \sqrt{42} \Rightarrow \quad PQ = 2PR = 2\sqrt{42}$

84.

Difficulty : Medium

Topics :

Math,
Vectors,

$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \Rightarrow |\hat{a}| = 3$

$\vec{b} = \hat{i} + \hat{j} \Rightarrow |\vec{b}| = \sqrt{2}$

$|\vec{c} - \vec{a}| = 3 \Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9 \qquad ...(1)$

$|(a \times b) \times \vec{c}\,| = 3 \Rightarrow |a \times b|\ |c|\ \sin\theta = 3$

$\Rightarrow 3 \times |c| \times \frac{1}{2} = 3 \Rightarrow \quad |c| = 2 \qquad ...(2)$

As $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \quad |\vec{a} \times \vec{b}| = 3$

Using (2) in (1) we have $4 + 9 - 2\ \vec{a} \cdot \vec{c} = 9$

$\Rightarrow \boxed{\vec{a} \cdot \vec{c} = 2}$

85.

Difficulty : Easy

Topics :

Math,
Probability,

n = number of trials = 10

p = probability of drawing green ball $= \frac{15}{25} = \frac{3}{5}$

= probability of success

q = probability of failure $= \frac{2}{5}$

As the trials are Bernoulli trials, variance = npq

$= 10 \cdot \frac{3}{5} \cdot \frac{2}{5}$

$= \boxed{\frac{12}{5}}$

86.

Difficulty : Medium

Topics :

Math,
Probability,

P(Exactly one of A or B occurs) $= P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) - 2P(A \cap B) = P(B) + P(C) - 2P(B \cap C)$

$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$

Adding all three $\Rightarrow \quad 2P(A) + 2P(B) + 2P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) = \frac{3}{4}$

P (Atleast one event occurs) = $P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cdot \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \boxed{\frac{7}{16}}$

87.

Difficulty : Medium

Topics :

Math,
Probability,

Two different numbers are taken from the set $\{0,1,2,.....10\} \Rightarrow |S| = {}^{11}C_2 = 55 \qquad ...(1)$

Let x + y = 4m .....(II)

x - y = 4n ..... (III), x > y

Now x = 2(m+n) $\Rightarrow$ y = 2(m-n)

Hence both x and y are even numbers

Then x and y are both in form of 4k or 4k + 2

So order pair of (x, y) are ${}^3C_2 + {}^3C_2 = 6 = |E|$

So Probability $= \dfrac{6}{55}$

88.

Difficulty : Medium

Topics :

Math,
Trigonometric Ratios & Identities,

$5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

Put $\cos 2x = t$

$\Rightarrow \quad 5\left[\dfrac{1-t}{1+t} - \dfrac{1+t}{2}\right] = 2t + 9$

$\Rightarrow \quad 5[2 - 2t - t^2 - 2t - 1] = (4t + 18)(t + 1)$

$\Rightarrow \quad 9t^2 + 42t + 13 = 0$

$\Rightarrow \quad 3t^2 + 3t + 39t + 13 = 0$

$\Rightarrow (3t + 13)(3t + 1) = 0$

$\Rightarrow \quad t = -\dfrac{1}{3}$

Now $\cos 4x = 2\cos^2 2x - 1 = \dfrac{2}{9} - 1 = \dfrac{-7}{9}$

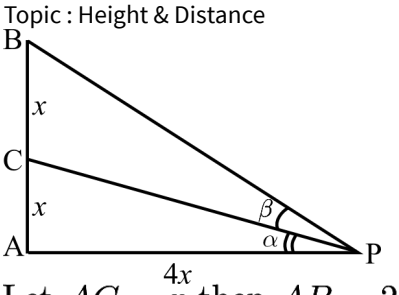
89.

Difficulty : Easy

Topics :

Math,

Topic : Height & Distance



Let $AC = x$ then $AB = 2x$ & $AP = 4x$
$\angle BPC = \beta$, Let $\angle CPA = \alpha$

Now $\tan(\alpha+\beta) = \frac{1}{2}$ and $\tan\alpha = \frac{1}{4}$

So $\tan\beta = \tan[(\alpha+\beta)-\alpha] = \frac{\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{2}\times\frac{1}{4}} = \frac{2}{9}$

90.

Difficulty : Easy

Topics :

Math,

Topic : Mathematical Reasoning

p	q	$x \equiv p \to q$	$\sim p$	$\sim p \to q$	$y \equiv (\sim p \to q) \to q$	$x \to y$
T	T	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

Hence it is a Tautology

Alternate :

$p \to q = \sim p \vee q$

$\sim p \to q = \sim(\sim p) \vee q = p \vee q$

$(\sim p \to q) \to q = \sim(\sim p \to q) \vee q = \sim(p \vee q) \vee q = (\sim p \wedge \sim q) \vee q$

$(p \to q) \to ((\sim p \to q) \to q) = \sim(p \to q) \vee ((\sim p \to q) \to q)$
$= \sim(\sim p \vee q) \vee (\sim p \wedge \sim q) \vee q$
$= (p \wedge \sim q) \vee (\sim p \wedge \sim q) \vee q$
$= t$