PHYSICS (QUESTION PAPER & SOLUTION)

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and it's mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 2.5 % (2) 3.5 % (3) 4.5 % (4) 6%

1. **(3)**

$d = \frac{M}{a^3}$

% Error is d = % Error is M + 3% Error is a

$= 1.5 + 3 \times 1$

$= 4.5$ %

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up

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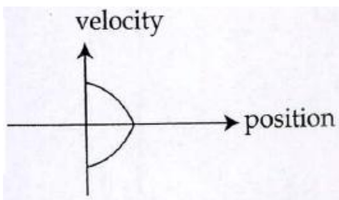

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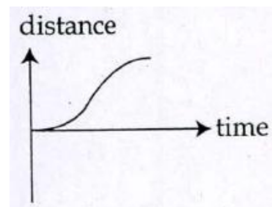

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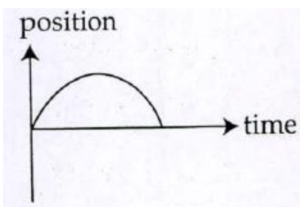

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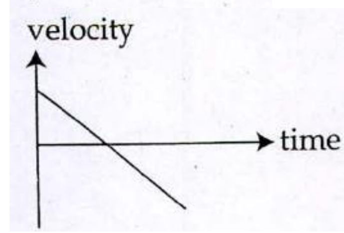
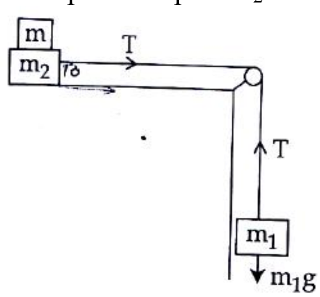


2. **(2)**

Graphs 1, 3, 4 represent vertical motion of an object under gravity.

3. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 18.3 kg (2) 27.3 kg (3) 43.3 kg (4) 10.3 kg

3. **(*)**

$\mu(m_2 + m)g \geq m_1 g$

$\Rightarrow (0.15)(10 + m)g \geq 5g$

$m \geq \frac{5}{0.15} - 10$

$m \geq 23.3$

4. A particle is moving in a circular path of radius a under the action of an attractive potential $U=-\frac{k}{2r^2}$. Its total energy is:

(1) $-\frac{k}{4a^2}$ (2) $\frac{k}{2a^2}$ (3) zero (4) $-\frac{3}{2}\frac{k}{a^2}$

4. **(1)**

$$\frac{dV}{dr}=\frac{-K}{2}\times\left(\frac{-2}{r^3}\right)$$

$$\frac{hV^2}{a}=\frac{k}{r^3}$$

$$hv^2=\frac{ak}{r^3}$$

$$KE=\frac{1}{2}hV^2=\frac{ak}{2r^3}$$

$$TE = KE + PE$$

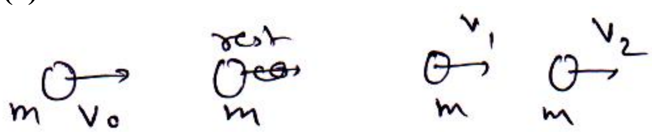
$$=\frac{-k}{2r^2}+\frac{ak}{2r^3} \quad \text{if } r=a$$

$$=0$$

5. A collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of he relative velocity between the two particles, after collision, is:

(1) $\frac{v_0}{4}$ (2) $\sqrt{2}\ v_0$ (3) $\frac{v_0}{2}$ (4) $\frac{v_0}{\sqrt{2}}$

5. **(2)**



$$mv_0=mv_1+mv_2$$

$$v_0=v_1+v_2 \quad \Rightarrow v_0^2=v_1^2+v_2^2+2v_1v_2$$

$$\frac{3}{2}\times\frac{1}{2}\times mv_0^2=\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2$$

$$\frac{3v_0^2}{2}=v_1^2+v_2^2$$

$$\frac{3v_0^2}{2}=\left(v_1+v_2\right)^2-2v_1v_2$$

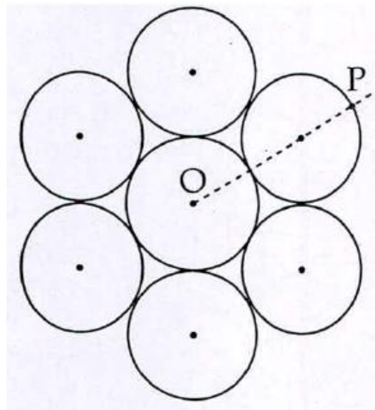
$$\frac{3v_0^2}{2}=v_0^2-2v_1v_2$$

$$2v_1v_2=-\frac{v_0^2}{2}$$

$$\left(v_1-v_2\right)^2=\frac{3v_0^2}{2}+\frac{v_0^2}{2}=2v_0^2$$

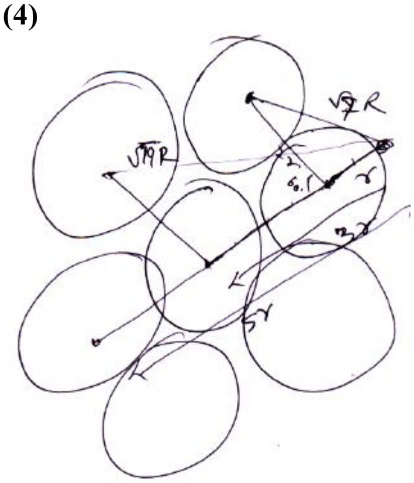
$$v_1-v_2=\sqrt{2}\,v_0$$

6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$ (3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

6. **(4)**

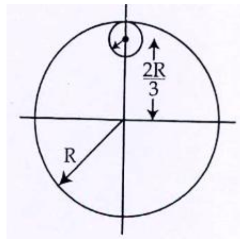


$$MR^2\left[\left(\frac{1}{2}+1\right)+\left(\frac{1}{2}+9\right)+\left(\frac{1}{2}+25\right)+\left(\frac{1}{2}+7\right)\times 2+\left(\frac{1}{2}+19\right)\times 2\right]$$

$$MR^2\left(\frac{3}{2}+\frac{19}{2}+\frac{51}{2}+15+39\right)$$

$$\frac{MR^2}{2}(3+19+51+30+78)=\frac{181MR^2}{2}$$

7. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



(1) $4\,MR^2$ (2) $\frac{40}{9}MR^2$ (3) $10\,MR^2$ (4) $\frac{37}{9}MR^2$

7. **(1)**

$$9\frac{MR^2}{2}-\left[\frac{M\left[R/3\right]^2}{2}+M\left(\frac{2R}{3}\right)^2\right]$$

$$\frac{9}{2}MR^2-\frac{MR^2}{18}+\frac{4MR^2}{9}=4MR^2$$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power R. If the period of rotation of the particle is T, then:

(1) $T\propto R^{3/2}$ for any n (2) $T\propto R^{\frac{n}{2}+1}$
(3) $T\propto R^{(n+1)/2}$ (4) $T\propto R^{n/2}$

8. **(3)**

$$m\omega^2R\propto\frac{k1}{R^n}$$

$$\omega^2\propto R^{n+1}$$

$$T=\frac{2\pi}{\omega}\Rightarrow T\propto R^{(n+1)}/2$$

9. A solid sphere of radius r made of soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is:

(1) $\frac{Ka}{mg}$ (2) $\frac{Ka}{3mg}$ (3) $\frac{mg}{3Ka}$ (4) $\frac{mg}{Ka}$

9. **(3)**

$$k=\frac{+dP}{dv/v} \qquad v=4/3\,\pi r^3$$

$$\frac{+\frac{mg}{a}}{\frac{dV}{V}}=k \qquad dv=4\pi r^2\,dr$$

$$\frac{dV}{V}=\frac{3dr}{r}$$

$$\frac{mg}{a\frac{3dr}{r}}=k$$

$$\frac{dr}{r}=\frac{mg}{3ak}$$

10. Two moles of an ideal monoatomic gas occupies a volume of V at 27^0C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) 2.7 kJ (2) (a) 195 K (b) – 2.7 kJ
(3) (a) 189 K (b) – 2.7 kJ (4) (a) 195 K (b) 2.7 kJ

10. **(3)**

$$PV^{\gamma}=\text{constant}$$

$$TV^{\gamma-1}=\text{constant}$$

$$300\times V^{2/3}=T_2(2V)^{2/3}$$

$$300=T_2\ 2^{2/3}$$

$$T_2=\frac{300}{4^{1/3}}=185$$

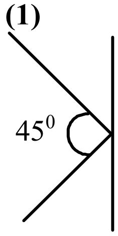
$$\Delta U=nc_V\Delta T$$

$$=2\times\frac{3R}{2}\times(185-300)=-2.7\,kJ$$

11. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area $2cm^2$ at an angle of 45^0 to the normal, and rebound elastically with a speed of $10^3 m/s$, then the pressure on the wall is hearly:

(1) $2.35\times10^3\ N/m^2$ (2) $4.70\times10^3\ N/m^2$ (3) $2.35\times10^2\ N/m^2$ (4) $7.40\times10^2\ N/m^2$

11. **(1)**



$$\Delta v=2v\cos45^0$$

$$F=10^{23}\times3.32\times10^{-27}\times2\times10^3\times\frac{1}{\sqrt{2}}$$

$$\text{Pressure}=\frac{F}{\text{Area}}=\frac{10^{23}\times3.32\times10^{-27}\times2\times10^3}{\sqrt{2}\times2\times10^{-4}}$$

$$=2.35\times10^3\ N/m^2$$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} / sec. What is the force constant of the bonds connecting one atom with the other? (mole wt. of silver = 108 and Avagadro number $= 6.02\times10^{23}$ gm mole^{-1})

(1) 6.4 N/m (2) 7.1 N/m (3) 2.2 N/m (4) 5.5 N/m

12. (2)

$k = mw^2 \qquad m = \frac{.108}{6.02\times10^{23}}$

$= \frac{.108}{6.02\times10^{23}}\times\left(2\pi\times10^{12}\right)^2$

$= \frac{.108\times4n^2\times10^{24}}{6.02\times10^{23}}$

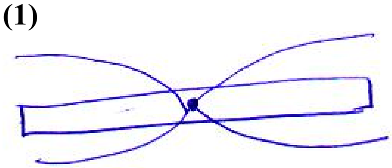
$= \frac{1.08\times4n^2}{6.02}$

= 7.1 N/m

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7\times10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 5 kHz (2) 2.5 kHz (3) 10 kHz (4) 7.5 kHz

13. (1)



$\lambda = 2\ell = 1.2m$

$v = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}$

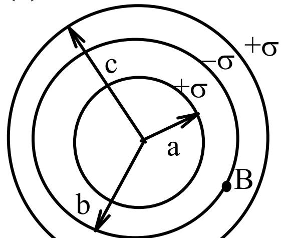
$= \frac{5.86\times10^3}{1.2} = 4.88\,KHZ$

= 5 KHZ

14. Three concentric metal shells A, P and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The [potential of shell B is:

(1) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (2) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (3) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+a\right]$ (4) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{c}+a\right]$

14. (2)



$V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{\sigma4\pi a^2}{b}-\frac{\sigma4\pi b^2}{b}+\frac{\sigma4\pi c^2}{c}\right)$

$$= \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right] = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

15. A parallel plate capacitor of capacitance 90 pF is connected to a batter of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :

(1) 1.2 n C (2) 0.3 n C (3) 2.4 n C (4) 0.9 n C

15. **(1)**

$C = \frac{5}{3} C_0 = 150\text{pF} = \text{new capacitance}$

$Q' = CV = 150\text{pF} \times 20\text{V}$

$Q_{ind} = Q'\left(1 - \frac{1}{k}\right) = 3\text{nC}\left(1 - \frac{3}{5}\right) = 1.2\,\text{nC}$

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\, t$

$i = 20 \sin\left(30\,t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattles current are, respectively :

(1) 50, 10 (2) $\frac{1000}{\sqrt{2}}, 10$ (3) $\frac{50}{\sqrt{2}}, 0$ (4) 50, 0

16. **(2)**

$E = 100 \sin(30t)$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

$z = 5$

$I_{wattless} = I_{rms}.\sin\phi$

$= \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

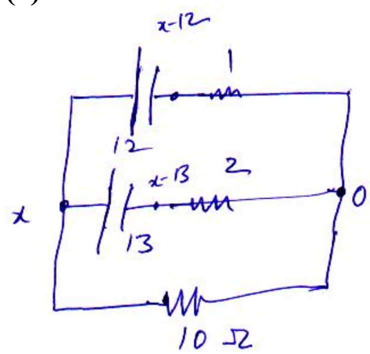
= 10

$\langle P \rangle = \frac{1000}{\sqrt{2}}$

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between:

(1) 11.6 V and 11.7 V (2) 11.5 V and 11.6 V
(3) 11.4 V and 11.5 V (4) 11.7 V and 11.8 V

17. **(2)**



$$\frac{x-12}{1}+\frac{x-13}{2}+\frac{x-0}{10}=0$$
$$\Rightarrow 10x-120+5x-65+x=0$$
$$\Rightarrow 16x=185 \quad \Rightarrow x=11.5\,V$$

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:

(1) $r_e > r_p = r_\alpha$ (2) $r_e < r_p = r_\alpha$ (3) $r_e < r_p < r_\alpha$ (4) $r_e < r_\alpha < r_p$

18. **(2)**

$$T=\frac{\sqrt{2mK}}{qB}$$

$$r_e : r_p : r_\alpha = \frac{\sqrt{m_e}}{q_e} : \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_\propto}}{q_\propto}$$

$$\Rightarrow r_e < r_p = r_\propto$$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

(1) 2 (2) $\sqrt{3}$ (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

19. **(3)**

$$m = I\times \pi R^2$$

$$B_1=\frac{\mu_0 I}{2R}$$

$$m^1 = 2m = I\times\pi\left(\sqrt{2}\,R\right)^2 = 2m$$

$$B_2=\frac{\mu_0 I}{2\left(\sqrt{2}\,R\right)}$$

$$\frac{B_1}{B_2}=\sqrt{2}$$

20. For an RLC circuit driven with voltage of amplitude υ_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exibits resonance. The quality factor, Q is given by:

(1) $\frac{\omega_0 L}{R}$ (2) $\frac{\omega_0 R}{L}$ (3) $\frac{R}{(\omega_0 C)}$ (4) $\frac{CR}{\omega_0}$

20. **(1)**

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\ \hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\ \hat{x}\cos\left[k(2z - ct)\right]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non – magnetic. If $\in_{r1}$ and $\in_{r2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\in_{r_1}}{\in_{r_2}} = 4$ (2) $\frac{\in_{r_1}}{\in_{r_2}} = 2$ (3) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$ (4) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$

21. **(3)**

$K_1 = \frac{2\pi V}{c}, \qquad w_1 = 2\pi v$

$K_2 = 2k, \qquad w_2 = kC$

$$\frac{v_{air}}{v_{medium}} = \sqrt{\frac{\in r_2}{\in r_1}}$$

$$\frac{w_1 | k_1}{w_2 | k_2} = \sqrt{\frac{\in r_2}{\in r_1}}$$

$$\frac{2k}{2\pi V/c} \times \frac{2\pi v}{kc} = \sqrt{\frac{\in r_2}{\in r_1}}$$

$$2 = \sqrt{\frac{\in r_2}{\in r_1}}$$

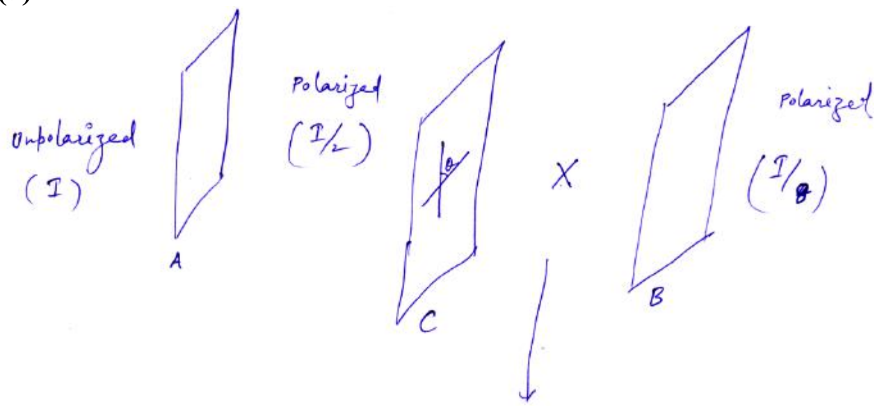
$$4 = \frac{\in r_2}{\in r_1}$$

$$\frac{\in r_1}{\in r_2} = \frac{1}{4}$$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 0^0 (2) 30^0 (3) 45^0 (4) 60^0

22. (3)



$x = \frac{I}{2}\cos^2\phi$ ...(1)

$\frac{I}{8} = \frac{I}{2} \times \cos^2\phi$ ...(2)

$\frac{I}{8} = \frac{I}{2}\cos^4\phi$

$\Rightarrow \frac{1}{4} = \cos^4\phi$

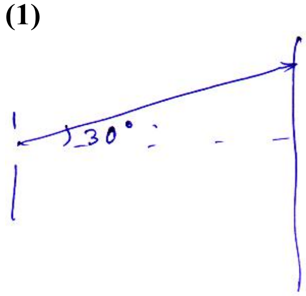
$\Rightarrow \cos\phi = \frac{1}{\sqrt{2}}$

$\Rightarrow \phi = 45^0$

23. The angular width of the central maximum in a single slit diffraction pattern is 60^0. The width of the slit is $1\,\mu m$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit).

(1) 25 μm (2) 50 μm (3) 75 μm (4) 100 μm

23. (1)



Angular width = 60^0

$\sin 30^0 = \frac{\lambda}{b} = \frac{\lambda}{10^{-6}}$

$\Rightarrow \lambda = 5\times10^{-7}$ m

Now, for Young's fringes,

$\phi = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times0.5}{d} = 10^{-2}$ m

$\Rightarrow d = \frac{5\times10^{-7}\times0.5}{10^{-2}} = 2.5\times10^{-5}$ m

$= 25\,\mu m$

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (2) $\Lambda_n \approx A + B\lambda_n$ (3) $\Lambda_n^2 \approx A + B\lambda_n^2$ (4) $\Lambda_n^2 \approx \lambda$

24. **(1)**

$$ke \propto \frac{1}{n^2}$$

$$\lambda_n \propto \sqrt{\frac{1}{2m(KE)}}$$

$$\Rightarrow \lambda_n \propto n \Rightarrow \lambda_n = \beta n \qquad \ldots(i)$$

Now,

$$\Lambda_n = \frac{hc}{-(E_g - E_n)} = \frac{hc}{+13.6\left(1 - \frac{1}{n^2}\right)}$$

$$\Rightarrow \Lambda_n = \frac{hc}{13.6\left(1 - \beta^2/\lambda_n^2\right)}$$

$$\Rightarrow \Lambda_n = \frac{hc}{13.6}\left(1 - \frac{\beta^2}{\lambda_n^2}\right)^{-1}$$

For $n \to \infty, \lambda_n \to \infty$ from (i)

$$\Lambda_n \approx \frac{hc}{13.6}\left(1 + \frac{\beta^2}{\lambda_n^2}\right)$$

$$\Rightarrow \Lambda_n = A + \frac{B}{\lambda_n^2}$$

25. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is:

(1) $25\nu_L$ (2) $16\nu_L$ (3) $\nu_L/16$ (4) $\nu_L/25$

25. **(4)**

$$v \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{v_2}{v_p} = \frac{\left(\frac{1}{1} - \frac{1}{\infty}\right)}{\left(\frac{1}{25} - \frac{1}{\infty}\right)}$$

$$\Rightarrow v_p = \frac{v_L}{25}$$

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively:

(1) $(\cdot 89, \cdot 28)$ (2) $(\cdot 28, \cdot 89)$ (3) (0, 0) (4) (0, 1)

26. **(1)**

$mv = mv_1 + 2mv_2$

$\Rightarrow v_1 + 2v_2 = v$ …(1)

From newton's

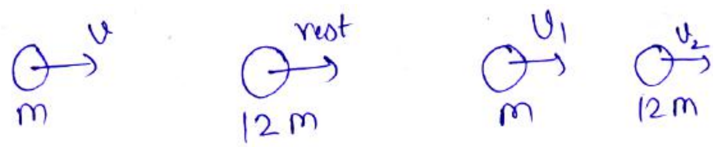
Law of collision $\Rightarrow \dfrac{v_2 - v_1}{0 - v} = -1$

$\Rightarrow v_2 - v_1 = v$ …(2)

$v_2 = \dfrac{2v}{3}, v_1 = -\dfrac{v}{3}$

Fractional loss = $= \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{3}\right)^2}{\frac{1}{2}mv^2}$

$= 1 - \dfrac{1}{9} = 0.89$



$mv = mv_1 + 12mv_2$

$\Rightarrow v_1 + 12v_2 = v$ …(1)

$\dfrac{v_2 - v_1}{0 - v} = -1$

$\Rightarrow v_2 - v_1 = v$ … (2)

From (1) & (2)

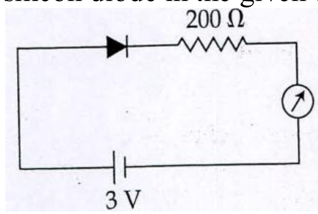
$v_1 + 12v + 12v_1 = v$

$v_1 = -\dfrac{11}{13}v$

$\text{loss} = \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2}$

$= 1 - \dfrac{121}{169} = 0.28$

27. The reading of the ammeter for a silicon diode in the given circuit is:



(1) 0 (2) 15 mA (3) 11.5 mA (d) 13.5 mA

27. **(2 or 3)**

If we consider diode to ideal

$$i=\frac{3}{200}=15\,\text{mA}$$

If we consider threshold voltage of silicon = 0.7 V

$$i=\frac{2.3}{200}=11.5\,\text{mA}$$

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^3 (2) 2×10^4 (3) 2×10^5 (4) 2×10^6

28. **(3)**

Only 10 % can be utilized.

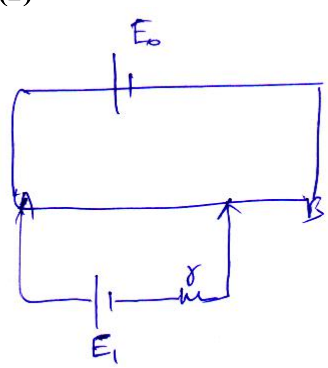
$$=\frac{\left(10\times10^9\right).\frac{10}{100}}{5\times10^3}$$

$$=2\times10^5$$

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of $5\,\Omega$, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) $1\,\Omega$ (2) $1.5\,\Omega$ (3) $2\,\Omega$ (4) $2.5\,\Omega$

29. **(2)**



$$\frac{.52}{1}=\frac{E_1}{V_{AB}}$$

$$\frac{.4}{1}=\frac{\frac{E_1\times5}{r+5}}{V_{AB}}$$

$$\frac{.52}{.4}=\frac{r+5}{5}$$
$$r=1.5\Omega$$

30. On interchanging the resistance, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is $1\,k\Omega$. How much was the resistance on the left slot before interchanging the resistances?

(1) $990\,\Omega$ (2) $505\,\Omega$ (3) $550\,\Omega$ (4) $910\,\Omega$

30. **(3)**

$$\frac{x}{100-x}=\frac{R_1}{R_2}$$
$$\frac{x-10}{90-x}=\frac{R_2}{R_1}$$
$$R_1+R_2=1000\,\Omega$$
$$\Rightarrow R_1=550$$
$$R_2=450$$

CHEMISTRY (QUESTION PAPER & SOLUTION)

31. The ratio of mass percent of C and H of an organic compound $\left(C_XH_YO_Z\right)$ is 6 : 1. If one molecule of the above compound $\left(C_XH_YO_Z\right)$ contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is:

(1) $C_3H_6O_3$ (2) C_2H_4O (3) $C_3H_4O_2$ (4) $C_2H_4O_3$

31. **(4)**

$\frac{12}{y}=\frac{6}{1}\Rightarrow y=2x$

$C_xH_y+\left(x+\frac{y}{4}\right)O_2\rightarrow xCO_2+\frac{y}{2}H_2O$

$Z=x+\frac{y}{4}=x+\frac{x}{2}=\frac{3x}{2}$

Considering $y=2x$ and $z=\frac{3x}{2}$

The compound most be $C_2H_4O_3$.

32. Which type of defect has the presence of cations in the interstitial sites?

(1) Schottky defect (2) Vacancy defect (3) Frenkel defect (4) Metal deficiency defect

32. **(3)**

When cations come from internitial site is called frenkel defect

33. According to molecular orbital theory, which of the following will **not** be a viable molecule?

(1) He_2^{2+} (2) He_2^{+} (3) H_2^{-} (4) H_2^{2-}

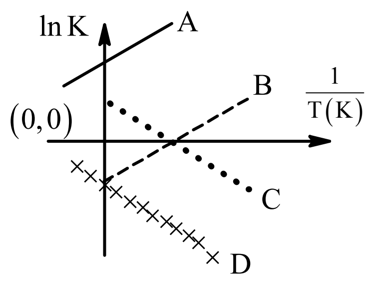
33. **(4)**

Those molecules which have zero band order is called viable molecule

$H_2{}^{2-}\longrightarrow \sigma ls^2,\sigma^* 1s^2$

$B.O=\frac{1}{2}\left(2-2\right)=0$

34. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?



(1) A and B (2) B and C (3) C and D (4) A and D

34. **(1)**

Slope $=\frac{-\Delta H}{RT}>0$ for exothermic reaction.

35. The combustion of benzene (l) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is $-3263.9\,kJ\ mol^{-1}$ at 25^o C; heat of combustion (in $kJ\ mol^{-1}$) of benzene at constant pressure will be:
$\left(R = 8.314\,JK^{-1}mol^{-1}\right)$
(1) 4152.6 (2) –452.46 (3) 3260 (4) –3267.6

35. **(4)**

$$C_6H_{6(\ell)} + \frac{15}{2}O_2(g) \rightarrow 6CO_{2(g)} + 3H_2O_{(\ell)}$$
$$\Delta H = \Delta U + \Delta ngRT$$
$$= (-3263.9) + (-1.5) \times \frac{8.314}{1000} \times 295\,kJ$$
$$= -3267.616\,kJ$$

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?
(1) $\left[Co(H_2O)_6\right]Cl_3$ (2) $\left[Co(H_2O)_5Cl\right]Cl_2.H_2O$
(3) $\left[Co(H_2O)_4Cl_2\right]Cl.2H_2O$ (4) $\left[Co(H_2O)_3Cl_3\right].3H_2O$

36. **(4)**
Smaller the number of particles provided by compound less will be the depression in freezing point and hence more will be the freezing point.

37. An aqueous solution contains 0.10 M H_2S and $0.20\,M\,HCl$. If the equilibrium constant for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is:
(1) 5×10^{-8} (2) 3×10^{-20} (3) 6×10^{-21} (4) 5×10^{-19}

37. **(2)**

$$K_1 = 1.0\times10^{-7} = \frac{0.2\times\left[HS^-\right]}{0.1}$$
$$\Rightarrow \left[HS^-\right] = 5\times10^{-8}M$$
$$K_2 = 1.2\times10^{-13} = \frac{0.2\times\left[S^{2-}\right]}{5\times10^{-8}}$$
$$\Rightarrow \left[S^{2-}\right] = 3\times10^{-20}\ M$$

38. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?
(1) $5\times10^{-9}M$ (2) $2\times10^{-9}M$ (3) $1.1\times10^{-9}M$ (4) $1.0\times10^{-10}M$

38. **(3)**
In final solution,

$$\left[Ba^{2+}\right]\times\frac{1}{10} = 10^{-10}$$
$$\Rightarrow \left[Ba^{2+}\right] = 10^{-9}$$

In original solution

$$\left[Ba^{2+}\right]=\frac{10^{-9}\times 500}{450}=1.1\times 10^{-9}$$

39. At $518^{o}C$, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was $1.00\,Torr\,s^{-1}$ when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is:

(1) 2 (2) 3 (3) 1 (4) 0

39. **(1)**

$$\frac{1}{0.5}=\left(\frac{0.95}{0.67}\right)^{x}$$

$$\Rightarrow (1.42)^{x}=2$$

$$\Rightarrow x=2$$

40. How long (approximate) should water be electrolyzed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?
(Atomic weight of $B=10.8\,u$)

(1) 6.4 hours (2) 0.8 hours (3) 3.2 hours (4) 1.6 hours

40. **(3)**

$$H_2O \longrightarrow O_2$$

(n – factor = 4)

$$B_2H_6+3O_2\rightarrow B_2O_3+3H_2O$$

$$\frac{100\times t\times 3600}{96500}=\frac{27.66}{27.6}\times 3\times 4 \mid t, \text{(in hours)}$$

$$\Rightarrow t=3.2\text{ hrs.}$$

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $\left[3Ca_3(PO_4)_2.Ca(OH)_2\right]$ to:

(1) $\left[CaF_2\right]$ (2) $\left[3(CaF_2).Ca(OH)_2\right]$

(3) $\left[3Ca_3(PO_4)_2.CaF_2\right]$ (4) $\left[3\{Ca(OH)_2\}CaF_2\right]$

41. **(3)**

42. Which of the following compounds contain(s) no covalent bond(s)?
$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

(1) KCl, B_2H_6, PH_3 (2) KCl, H_2SO_4

(3) KCl (4) KCl, B_2H_6

42. **(3)**

KCl exist as $K^{+}Cl^{-}$

43. Which of the following are Lewis acids?

(1) PH_3 and BCl_3 (2) $AlCl_3$ and $SiCl_3$

(3) PH_3 and $SiCl_4$ (4) BCl_3 and $AlCl_3$

43. **(4)**
B & Al are group 13 electrons and BCl_3 & $AlCl_3$ are electron deficient

44. Total number of lone pair of electrons in I_3^- ion is:
(1) 3 (2) 6 (3) 9 (4) 12

44. **(3)**
$:\ddot{\underset{\cdot\cdot}{I}} - \ddot{\underset{\cdot\cdot}{I}} - \ddot{\underset{\cdot\cdot}{I}}:$ (central I carries three lone pairs)

45. Which of the following salts in the most basic in aqueous solution?
(1) $Al(CN)_3$ (2) CH_3COOK (3) $FeCl_3$ (4) $Pb(CH_3COO)_2$

45. **(2)**

46. Hydrogen peroxide oxidizes $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)^6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively :
(1) (H_2O_2) and H_2O (2) (H_2O+O_2) and (H_2O+OH^-)
(3) H_2O and (H_2O+O_2) (4) H_2O and (H_2O+OH^-)

46. **(3)**
$$2K_3\left[Fe(CN)_6\right] + 2KOH + H_2O_2 \longrightarrow 2K_4\left[Fe(CN)_6\right] + O_2 + 2H_2O$$

47. The oxidation states of Cr in $\left[Cr(H_2O)_6\right]Cl_3, \left[Cr(C_6H_6)_2\right]$, and $K_2\left[Cr(CN)_2(O)_2(O)_2(NH_3)\right]$ respectively are :
(1) +3, +4 and +6 (2) +3, +2 and +4 (3) +3, 0 and +6 (4) +3, 0 and +4

47. **(3)**
$$\left[Cr^{+3}\left(H_2^0O\right)_6\right]^{-1} Cl_3 \left[Cr^0\left(C_6{}^0H_6\right)_2\right] \quad K_2^{+1}\left[Cr^{+6}\left(CN^{-1}\right)_2 (O)_2^{-2}\left(N^oH_3\right)\right]$$
$$H_2O_2 + 2K_4\left[Fe(CN)_6\right] + H_2SO_4 \rightarrow 2K_3\left[Fe(CN)_6\right] + K_2SO_4 + 2H_2O$$

48. The compound that **does not** produce nitrogen gas by the thermal decomposition is :
(1) $Ba(N_3)_2$ (2) $(NH_4)Cr_2O_7$ (3) NH_4NO_2 (4) $(NH_4)_2SO_4$

48. **(4)**
$$(NH_4)_2SO_4 \rightarrow NH_3 + H_2SO_4$$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :
(1) Zn (2) Ca (3) Al (4) Fe

49. **(3)**
$$Al + 3OH^- \rightarrow Al(OH)_3 \xrightarrow[\text{excess}]{} \left[Al(OH)_6\right]^{-3} \xrightarrow{\Delta} \underset{\substack{\text{(Alumina)} \\ \text{(Used in column chromatography)}}}{Al_2O_3}$$

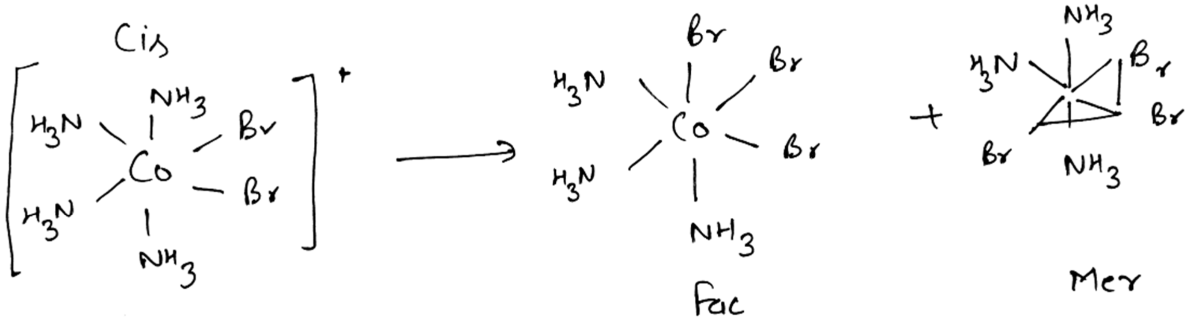
50. Consider the following reaction and statement :.

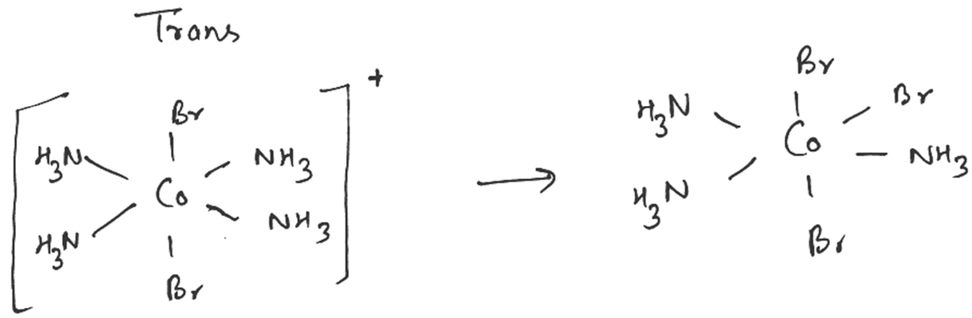
$\left[Co(NH_3)_4 Br_2\right]^+ + Br^- \rightarrow \left[Co(NH_3)_3 Br_3\right] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.
(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.
(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.
The correct statements are :
(1) (I) and (II) (2) (I) and (III) (3) (III) and (IV) (4) (II) and (IV)

50. **(2)**

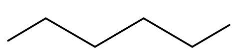
$\left[Co(NH_3)_4 Br_2\right]^+ + Br^- \rightarrow \left[Co(NH_3)_3 Br_3\right] + NH_3$





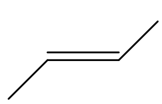
51. Glucose on prolonged heating with HI gives :
(1) *n*-Hexane (2) 1-Hexene (3) Hexanoic acid (4) 6-iodohexanal

51. **(1)**

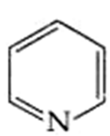
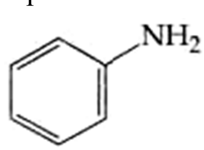
Glocose $\xrightarrow[\Delta]{HI}$  Hexane

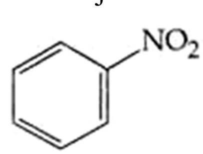
52. The *tans*-alkenes are formed by the reduction of alkynes with :
(1) H_2 – Pd/C, $BaSO_4$ (2) $NaBH_4$
(3) Na/liq. NH_3 (4) Sn – HCl

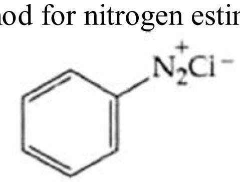
52. **(3)**

 $\xleftarrow{Na/lq/Na}$ Birch reduction

53. Which of the following compound will be suitable for Kjeldahl's method for nitrogen estimation?

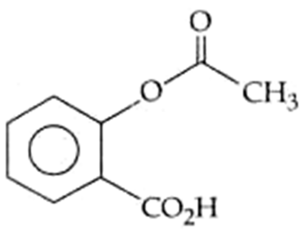
(1)  (2) 

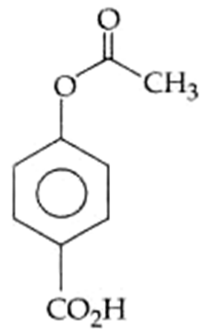
(3) 

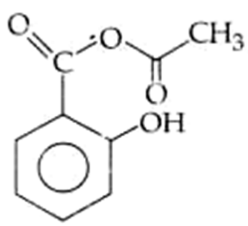
(4) 

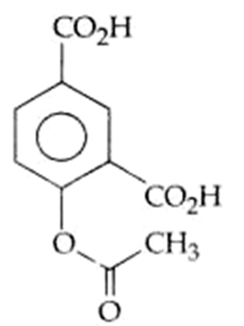
53. **(2)**

54. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compounds X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

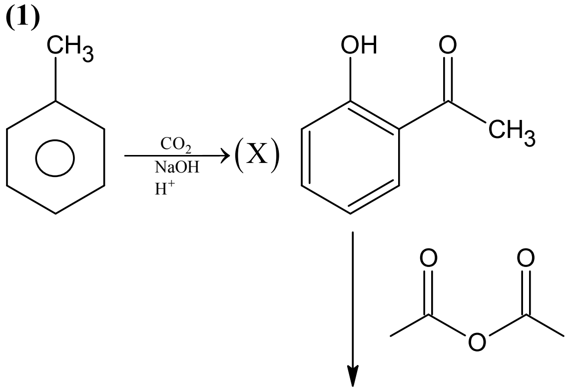
(1) 

(2) 

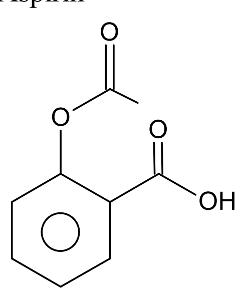
(3) 

(4) 

54. **(1)**



Aspirin



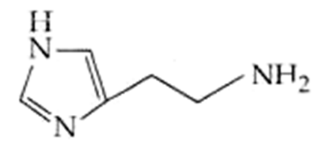
55. An alkali is tritated against an acid with methyl orange as indicator, which of the following is a correct combination?

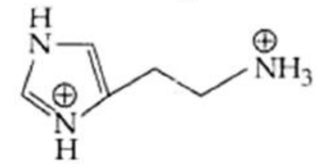
	Base	Acid	End point
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to pinkish red
(4)	Strong	Strong	Pink to colourless

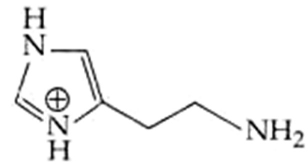
55. (3)

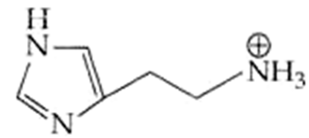
Weak base when titrated with strong acid, the equivalent point comes in acidic range pH.
Hence the color of solution will be changed from yellow to pinkish red.

56. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

(1) 

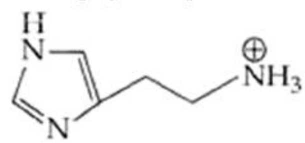
(2) 

(3) 

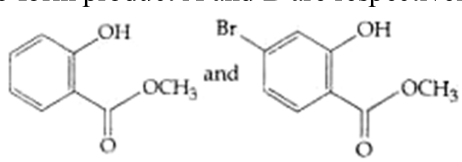
(4) 

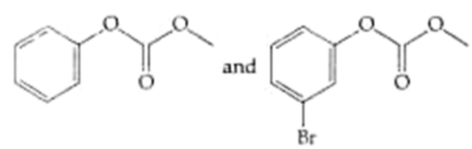
56. (4)

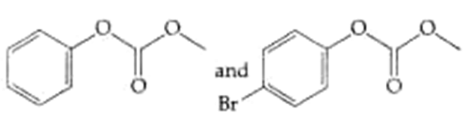
Under physiological condition aliphatic amines get protonate.

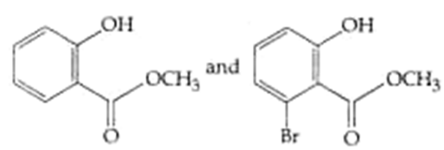


57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product A and B are respectively:

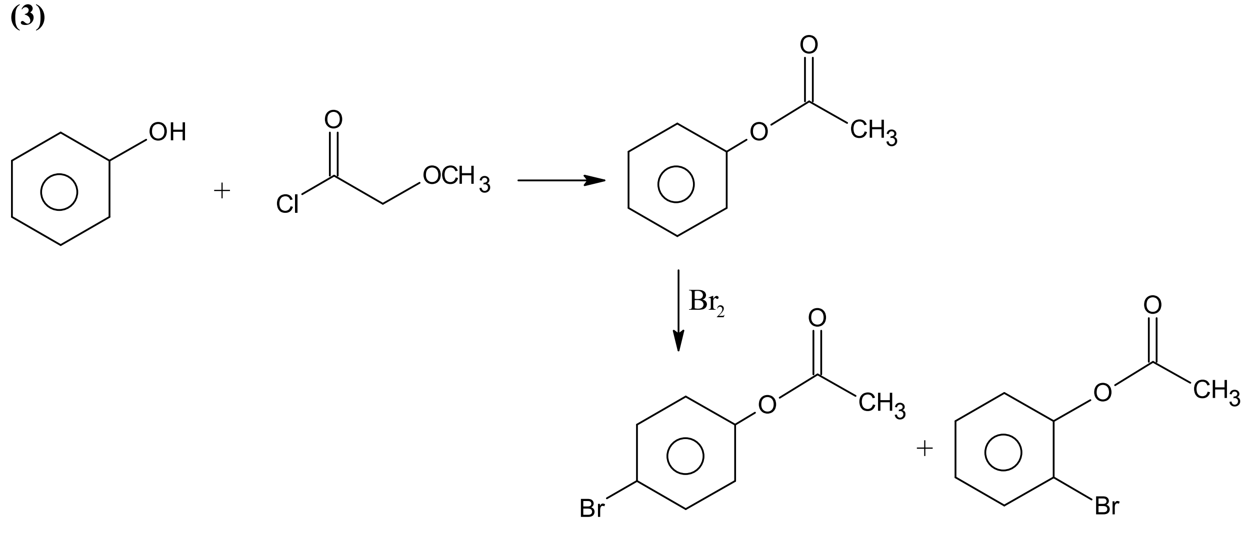
(1) 

(2) 

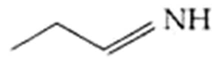
(3) 

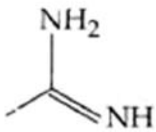
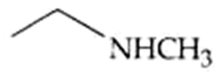
(4) 

57. (3)



58. The increasing order of basicity of the following compound is :

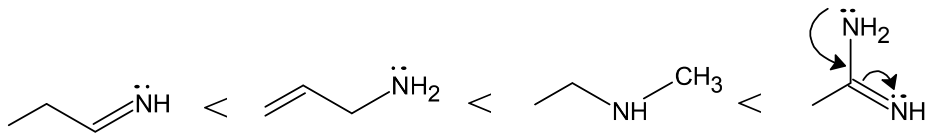
(a)  (b) 

(c)  (d) 

(1) (a) < (b) < (c) < (d) (2) (b) < (a) < (c) < (d)

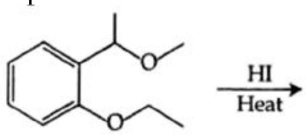
(3) (b) < (a) < (d) < (c) (4) (d) < (b) < (a) < (c)

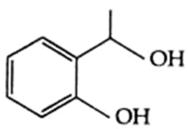
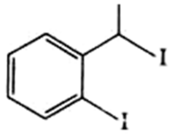
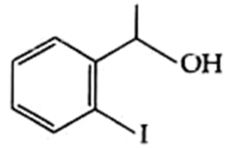
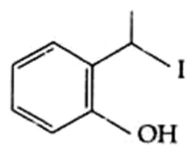
58. **(3)**



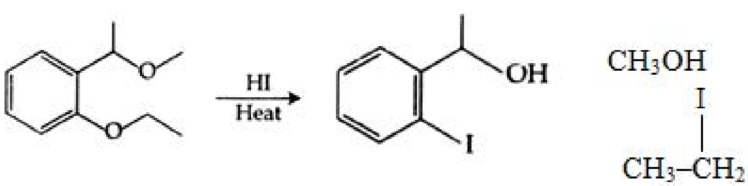
Basic order

59. The major product formed in the following reaction is :

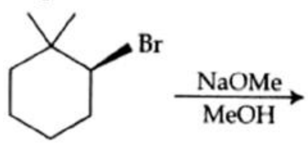


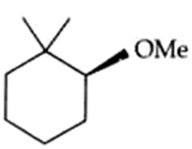
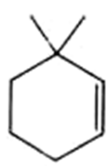
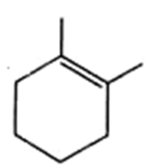
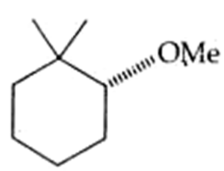
(1)  (2)  (3)  (4) 

59. **(4)**

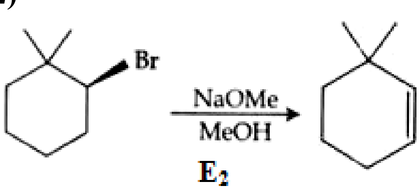


60. The major product of the following reaction is :



(1)  (2)  (3)  (4) 

60. **(2)**



MATHEMATICS (QUESTION PAPER & SOLUTION)

61. Two Sets A and B are as under

$A\{(a,b)\}\in:|a-5|<1 \text{ and } |b-5|<1\}$;

$B=\{(a,b)\in R\times R: 4(a-6)^2+9(b-5)^2\le 36\}$. Then:

(1) $B\subset A$ (2) $A\subset B$

(3) $A\cap B=\phi$ (en empty set) (4) neither $A\subset B$ nor $B\subset A$

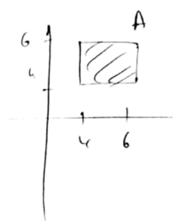
61. **(2)**

In Set A

$-1<a-5<1$

$4<a<6$ $a\in(4,6)$

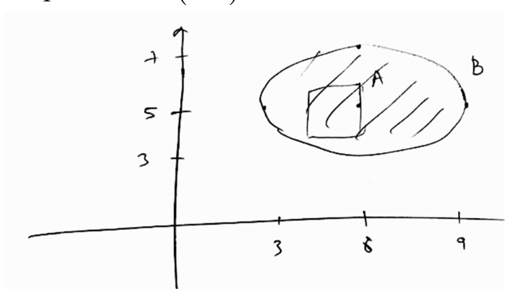
$b\in(4,6)$



In set B

$\& \frac{(a-6)^2}{9}+\frac{(b-5)^2}{4}\le 1$

Ellipse centre $(6,5)$



62. Let $S=\{x:R:x\ge 0 \text{ and } 2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S:

(1) is an empty set (2) contains exactly one element

(3) contains exactly two elements (4) contains exactly four elements

62. **(3)**

$2|\sqrt{x}-3|+(\sqrt{x})^2-6\sqrt{x}+9-9+6=0$

$2|t|+t^2-3=0$

$t^2+2|t|-3=0$

$|t|=1,-3$

$t=\pm 1$

$\sqrt{x}-3=\pm 1$

$\sqrt{x}=4 \text{ or } 2$

$x=16 \text{ or } 4$

63. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(1) -1 (2) 0 (3) 1 (4) 1

63. **(3)**

$x^2 - x + 1 = 0$

$x = -\omega, -\omega^2$

So $\alpha = -\omega$

$\beta = -\omega^2$

$\alpha^{101} + \beta^{107}$

$= (-\omega)^{101} + (-\omega^2)^{107}$

$= -(\omega^2 + \omega)$

$= +1$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$ then the ordered pair (A, B) is equal to:

(1) $(-4, -5)$ (2) $(-4, 3)$ (3) $(-4, 5)$ (4) $(4, 5)$

64. **(3)**

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$R_3 = R_3 - R_1$

$R_2 = R_2 - R_1$

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 0 \\ 0 & 0 & -x-4 \end{vmatrix}$$

$= (5x-4)(x+4)^2$

$= (-4+5x)(x-(-47))^2$

$A = -4, B = 5$

65. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

Has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to:

(1) -10 (2) 10 (3) -30 (4) 30

65. **(2)**

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$(-3k+8)-k(-9+4)+3(12-2k)=0$

$-3k+8+5k+36-6k=0$

$44-4k=0$

$k=11$

$x+11y+3z=0 \quad \ldots\ldots(1)$

$3x+11y-2z=0 \quad \ldots\ldots(2)$

$3x+33y+9z=0 \quad \ldots\ldots(3)$

$-22y-11z=0$

$z=-2y$

$x+11y-6y=0$

$x+5y=0$

$x=-5y$

$$\frac{(-5y)(-2y)}{y^2}=10$$

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

(1) at least 1000 (2) less than 500

(3) at least 500 but less than 750 (4) at least 750 but less than 1000

66. **(1)**

${}^6C_4 \times {}^3C_1 \times 4!$

$= {}^6C_2 . 3 \times 24$

$= 15.3 \times 24$

$= 15 \times 72 = 720 + 360 = 1080$

67. The sum of the co-efficient of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x>1)$ is:

(1) -1 (2) 0 (3) 1 (4) 2

67. **(4)**

$(a+b)^5+(a-b)^5$

$= a^5$	a^5
$5a^4b$	$-5a^4b$
$10a^3b^2$ +	$10a^3b^2$
$10a^2b^3$	$-10a^2b^3$
$5ab^4$	$+5ab^4$
b^5	$-b^5$

$=2\left[a^5+10a^3b^2+5ab^4\right]$

$=2\left[x^5+10x^3\left(x^3-1\right)+5x\left(x^3-1\right)^2\right]$

$= 2\left[x^5 + 10x^6 - 10x^3 + 5x\left(x^6 - 2x^3 + 1\right)\right]$

$= 2x^5 + 20x^6 - 20x^3 + 10x^7 - 20x^4 + 10x$

$2 - 20 + 10 + 10 = 2$

68. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to:

(1) 66 (2) 68 (3) 34 (4) 33

68. **(3)**

Let the AP has first term a and common difference d

So the equation $\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a + 24d = 32$ and equation $a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$

Solving these two equations $a = 8$ and $d = 1$

So $\sum_{k=1}^{17} a_k^2 = 140m \Rightarrow m = 34$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots$ If $B - 2A = 100\lambda$, then λ is equal to:

(1) 232 (2) 248 (3) 464 (4) 496

69. **(2)**

$A = 1^2 + 2.2^2 + 3^2 + 2.4^2 \ldots\ldots + 19^2 + 2.20^2$

$= 1^2 + 2^2 + 3^2 \ldots\ldots\ldots + 20^2$

$\quad + 2^2 + 4^2 \ldots\ldots + 20^2$

$= \frac{20 \times 21 \times 41}{6} + 2^2 \times \frac{10 \times 11 \times 21}{6}$

$= 41 \times 70 + 1540$

$= 2870 + 1540 = 4410$

$B = 1^2 + 2^2 \ldots\ldots + 40^2 + 2^2 + 4^2 \ldots\ldots + 40^2$

$= \frac{40 \times 41 \times 81}{6} + 4 \times \frac{20 \times 21 \times 41}{6}$

$= 22140 + 11480 = 33620$

$B - 2A = 24800$

70. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$\lim_{x \to 0} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

(1) Is equal to 0 (2) is equal to 15 (3) is equal to 120 (4) does not exist (in R)

70. **(3)**

We know

$x - 1 < [x] \le x$

So, $\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \le \frac{1}{x}$

$\frac{2}{x} - 1 < \left[\frac{2}{x}\right] \le \frac{2}{x}$

.
.
.
.
.
.

$\frac{15}{x}-1<\left[\frac{15}{x}\right]\le\frac{15}{x}$

Add

$\frac{15\times 8}{x}-15<\sum_{r=1}^{15}\left[\frac{r}{x}\right]\le\frac{15\times 8}{x}$

$15\times 8-15x<x\sum_{r=1}^{15}\left[\frac{r}{x}\right]\le 15\times 8$

Now by sandwich theorem $\ell t\, x\to 0^{+}$

$15\times 8<\lim_{x\to 0^{+}} x\sum_{r=1}^{15}\left[\frac{r}{x}\right]\le 15\times 8$

71. Let $S=\{t\in R: f(x)=|x-\pi|.(e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to:

(1) ϕ (an empty set) (2) $\{0\}$ (3) $\{\pi\}$ (4) $\{0,\pi\}$

71. **(1)**

Derivability is questionable at

$x=\pi$ & $x=0$

At $x=0$

$f'(0)=\lim_{h\to 0}\frac{|h-\pi|(e^{|h|}-1)\sin|h|}{h}$

$=0$

At $x=\pi$ $f'(\pi)=\lim_{h\to 0}\frac{|h|(e^{|\pi+h|}-1)\sin|h|}{h}$

$=0$

$\Rightarrow$ Differentiable everywhere

72. If the curves $y^2=6x, 9x^2+by^2=16$ intersect each other at right angles, then the value of b is:

(1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

72. **(4)**

Given curves are :

$y^2=6x; 9x^2+by^2=16$

$y^2=6x\to$ differentiating with respect to x :

$2y.\frac{dy}{dx}=6$

$\frac{dy}{dx}=\frac{3}{y}$

$9x^2+by^2=16\to$ diff.wrt x

$18x+2by.\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-9x}{by}$

If the curves intersect orthogonally, then

$\left.\frac{dy}{dx}\right|_{I}.\left.\frac{dy}{dx}\right|_{II}=-1$

$\Rightarrow \quad \frac{3}{y}\times\left(\frac{-9x}{by}\right)=-1 \quad \Rightarrow \quad 27x=by^2$

$\Rightarrow \quad y^2=\left(\frac{27}{b}\right)x$

Comparing : $\frac{27}{b}=6 \Rightarrow \quad b=\frac{9}{2}$

73. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}, x\in R-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

73. **(4)**

$f(x)=x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+2$

$g(x)=x-\frac{1}{x}$

$\therefore h(x)=\frac{f(x)}{g(x)}=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$

$\therefore$ local minimum $=2\sqrt{2}$

74. The integral

$\int\frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x\right)^2}dx$ is equal to:

(1) $\frac{1}{3\left(1+\tan^3 x\right)}+C$ (2) $\frac{-1}{3\left(1+\tan^3 x\right)}+C$

(2) $\frac{1}{1+\cot^3 x}+C$ (4) $\frac{-1}{1+\cot^3 x}+C$

(where C is a constant of integration)

74. **(2)**

By manipulation the integration can be simplified as $I=\int\frac{\tan^2 x\sec^2 x\,dx}{\left(1+\tan^3 x\right)^2}$

By substituting $1+\tan^3 x=t$

We get the integration as $I = \frac{-1}{3} \cdot \frac{1}{(1+\tan^3 x)} + c$

75. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$ (3) 4π (4) $\frac{\pi}{4}$

75. (4)

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}$ ------(1)

$I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1+2^x} dx$ ------(2)

$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x dx = 2\int_0^{\pi/2} \sin^2 x\, dx$

$I = \int_0^{\pi/2} \sin^2 x dx$

$I = \int_0^{\pi/2} \cos^2 x dx$

$2I = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

76. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$ and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is:

(1) $\frac{1}{2}(\sqrt{3}-1)$ (2) $\frac{1}{2}(\sqrt{3}+1)$ (3) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ (4) $\frac{1}{2}(\sqrt{2}-1)$

76. (1)

$g(x) = \cos x^2$

$f(x) = \sqrt{x}$

$g(f(x)) = \cos x$ $18x^2 - 9\pi x + \pi^2 = 0$

Roots: $\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

$\therefore \text{Area} = \int_{\pi/6}^{\pi/3} \cos x\, dx$

$= \frac{1}{2}(\sqrt{3}-1)$

77. Let $y = y(x)$ be the solution of the differential equation

$\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{4}{9\sqrt{3}}\pi^2$ (2) $\frac{-8}{9\sqrt{3}}\pi^2$ (3) $-\frac{8}{9}\pi^2$ (4) $-\frac{4}{9}\pi^2$

77. **(3)**

$\frac{dy}{dx}+\cot x\, y=\frac{4x}{\sin x}$

$I.F.=e^{\int \cot dx}=e^{\log \sin x}=\sin x$

$y=\sin x=\int \frac{4x}{\sin x}.\sin x$

$y\sin x=\frac{4n^2}{2}+c$

$y\times \sin\left(\frac{\pi}{2}\right)=x-\frac{\pi^2}{4}+c$

$\therefore c=-\frac{\pi^2}{2}$

$y\sin x=2x^2-\frac{\pi^2}{2}$

$y\times \sin\left(\frac{\pi}{6}\right)=2\times\frac{\pi^2}{36}-\frac{\pi^2}{2}$

$y.\frac{1}{2}=\frac{\pi^2}{2}\left(\frac{1}{9}-1\right)=\frac{\pi^2}{z}\times\frac{-8}{9}$

$y=\frac{-\pi^2}{9}.8$

78. A straight line through a fixed point $(2,3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed , then the locus of R is:

(1) $3x+2y=6$ (2) $2x+3y=xy$ (3) $3x+2y=xy$ (4) $3x+2y=6xy$

78. **(3)**

Let line be

$\frac{x}{a}+\frac{y}{b}=1$

Passes through (2,3)

$\therefore \frac{2}{x}+\frac{3}{y}=1$

$\Rightarrow 3x+2y=xy$

79. Let the orthocenter and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter , is:

(1) $\sqrt{10}$ (2) $2\sqrt{10}$ (3) $3\sqrt{\frac{5}{2}}$ (4) $\frac{3\sqrt{5}}{2}$

79. **(3)**

Orthocentre $(-3,5)$

Centroid (3,3)

By euler line

Circumcentre : $\frac{2x-3}{3}=3 \Rightarrow x=6$

$\& \frac{2y+5}{3}=3 \Rightarrow y=2$

$\therefore (6,2)$

$\therefore$ Radius : $\sqrt{\frac{81+5}{4}}=3\sqrt{\frac{5}{2}}$

80. If the tangent at $(1,7)$ to the curve $x^2=y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is:

(1) 195 (2) 185 (3) 85 (4) 95

80. **(4)**

Tangent at $(1,7)$ to the parabola $x^2=y-6$ is $2x-y+5=0$

Now circle is $(x+8)^2+(y+6)^2=100-c$

For line to be tangent distance of line from centre is equal to radius

$\therefore c=95$

81. Tangent and normal are drawn at P(16,16) on the parabola $y^2=16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points A and B and $\angle CPB=\theta$, then a value of $\tan\theta$ is:

(1) $\frac{1}{2}$ (2) 2 (3) 3 (4) $\frac{4}{3}$

81. **(2)**

Equations of tangent and normal to parabola $y^2=16x$ at $(16,16)$ are

$x-2y+16=0,\ 2x+y-48=0$ respectively

So the points are $A(-16,0)$, $B(24,0)$

As PAB is a right triangle midpoint of AB is the centre of the circle

$\therefore$ C=(4,0)

Hence Slope of PC is 4/3 and slope BP is -2

Then $\tan\theta=2$

82. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point $T(0,3)$ then the area (in sq. units) of ΔPTQ is:

(1) $45\sqrt{3}$ (2) $54\sqrt{3}$ (3) $60\sqrt{3}$ (4) $36\sqrt{5}$

82. **(1)**

$y=mx\pm\sqrt{a^2m^2-b^2}$

$3=\pm\sqrt{9m^2-36}$

$9=9m^2-36$

$9m^2=45$

$m^2=5$

$m=\pm\sqrt{5}$

Equation of tangent at point (x_1,y_1) is $4xx_1-yy_1=36$

Point P

$4xx_1 - yy_1 = 36$

$-12\sqrt{5}x + 12y = 36$

$x_1 = -3\sqrt{5}$

$y_1 = -12$

Point Q

$4xx_1 - yy_1 = 36$

$\sqrt{5x} + y = 3$

$12\sqrt{5x} + 12y = 36$

$4x_1 = 12\sqrt{5}$

$x_1 = 3\sqrt{5}$

$y_1 = -12$

Tangents are

$y - 3 = \sqrt{5}x$........P

$y - 3 = -\sqrt{5}x$........Q

$\sqrt{5}x - y = -3$

$-12\sqrt{5}x + 12y = 36$

83. If L_1 is the line of intersection of the plane $2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0$ are L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is:

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

83. **(2)**

Using concept of family of planes

Plane containing L_1

$2x - 2y + 3z - 2 + \alpha(x - y + z + 1) = 0$

And

Plane containing L_2

$x + 2y - z - 3 + \beta(3x - y + 2z - 1) = 0$

Since these 2 are same planes, by comparing we get

$$\frac{2+\alpha}{3\beta+1} = \frac{-2-\alpha}{2-\beta} = \frac{3+\alpha}{-1+2\beta} = \frac{2-\alpha}{3+\beta}$$

By solving these

$\beta = -3/2, \alpha = 5$

We get equation of plane as $-7x + 7y - 8z = 3$

Hence perpendicular distance from origin is $\frac{1}{3\sqrt{2}}$

84. The length of the projection of the line segment joining the points $(5,-1,4)$ and $(4,-1,3)$ on the plane $x+y+z=7$ is:

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\sqrt{\frac{2}{3}}$

84. **(4)**

$\overrightarrow{AB}=\hat{i}+\hat{k}$

$\vec{n}=\hat{i}+\hat{j}+\hat{k}$

Required projection $\left|\overrightarrow{AB}\times\hat{n}\right|$

$=\sqrt{\frac{2}{3}}$

85. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to a and $\vec{u}.\vec{b}=24$, then $\left|\vec{u}\right|^2$ is equal to:

(1) 336 (2) 315 (3) 256 (4) 84

85. **(1)**

Let $\vec{u}=\lambda(2i+3j-k)+y(j+k)$

Using the conditions $\vec{u}.\vec{a}=0$, and $\vec{u}.\vec{b}=24$ we get $\lambda=-2,\ y=14$

So $\vec{u}=-4i+8j+16k$

$\Rightarrow\left|\vec{u}\right|=\sqrt{336}$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{3}{10}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{3}{4}$

86. **(2)**

Case I

First ball red has probability $=\frac{4}{10}$

Now the bag has $6R\,\&\,6B$

Then probability of next being red is $=\frac{6}{12}$

Case II

First ball black has probability $=\frac{6}{10}$

Now the bag has 4 R & 8B balls

Probability of next being red is $\frac{4}{12}$

Hence total probability is $\frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}=\frac{2}{5}$

87. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1,x_2,......,x_9$ is:

(1) 9 (2) 4 (3) 2 (4) 3

87. **(3)**

S.D. is independent of shifting of origin

$\text{Variance} = E(x^2) - (E(x))^2$

$\text{S.D.} = \sqrt{4} = 2$

88. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to:

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$ (3) $\frac{8}{9}$ (4) $\frac{20}{9}$

88. **(2)**

We know $\cos(A+B)\cos(A-B) = \cos^2 B - \sin^2 A$

The equation simplifies to $\cos 3x = \frac{1}{2}$

$x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

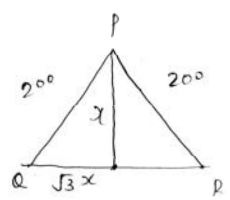
The solution in the range $[0,\pi]$ are $\frac{\pi}{9}, \frac{7\pi}{9}, \frac{5\pi}{9}$

89. PQR is a triangular park with $PQ = PR = 200\text{m}$. A.T.V tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P,Q and R respectively $45^\circ, 30^\circ$ and 30°, then the height of the tower (in m) is:

(1) 100 (2) 50 (3) $100\sqrt{3}$ (4) $50\sqrt{2}$

89. **(1)**

Sol:



$x^2 + 3x^2 = (200)^2$

$4x^2 = (200)^2$

$x = 100$

90. The Boolean expression

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

(1) $\sim p$ (2) p (3) q (4) $\sim q$

90. **(1)**

$\sim(p \vee q) \vee (`p \wedge q)$

$= \sim((p \vee q) \wedge (p \vee \sim q))$

$= \sim((p \text{ or } q) \& (p \text{ or not } q))$

$= \sim(p)$

$= \sim p$

p	q	$p \vee q$	$\sim (p \vee q)$	$\sim p$	$\sim p \wedge q$	$\sim (p \vee q) \vee (\sim p \wedge q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T