CODE-A
PHYSICS

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :
   1) 2.5% 2) 3.5% 3) 4.5% 4) 6%

Key: (3)

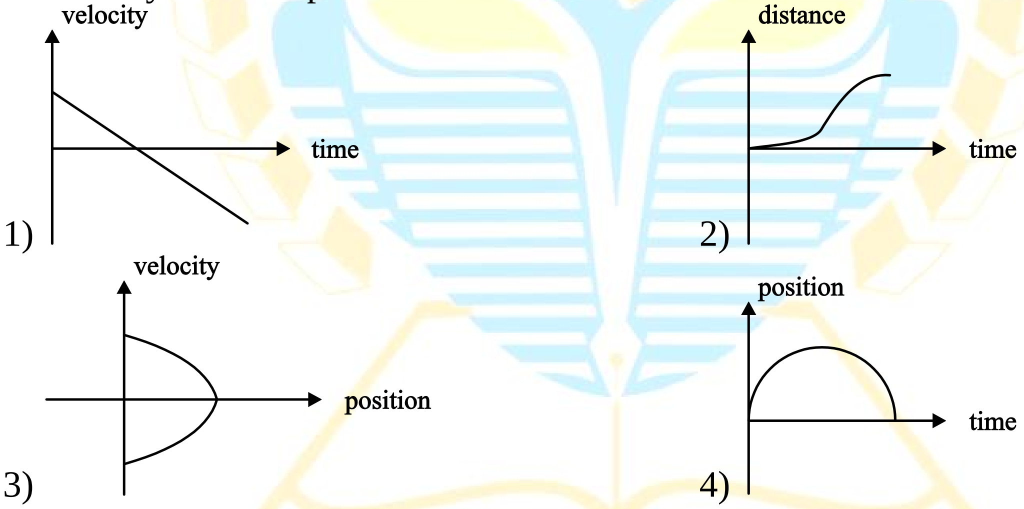
Sol: density $= \frac{M}{V}$

$\frac{\Delta p}{p} = \frac{\Delta M}{V} + 3\frac{\Delta r}{r}$

= 1.5% + 3 %

= 4.5 %

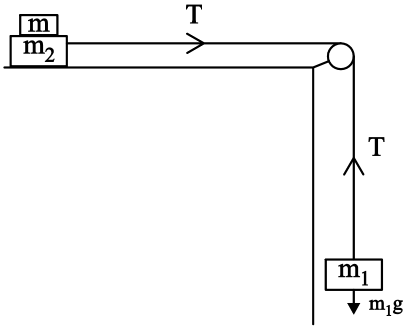
2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



Key: (2)

Sol: Initial and final slopes are zero (so option '2' is wrong) all other are correct

3. Two masses $m_1 = 5kg$ and $m_2 = 10kg$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :



1) 18.3 kg 2) 27.3 kg 3) 43.3 kg 4) 10.3 kg

Key: (2)

Sol: $\leftarrow \boxed{m_2} \rightarrow 50N$

$\mu(m_2+m)g$

For equilibrium. 0.15[10 + m]10 = 50

$10 + m = \frac{500}{15}$

M = 23.33 kg

4. A particle is moving in a circular path of radius a under the action of an attractive potential $U=-\frac{k}{2r^2}$. Its total energy is :

1) $-\frac{k}{4a^2}$ 2) $\frac{k}{2a^2}$ 3) Zero 4) $-\frac{3}{2}\frac{k}{a^2}$

Key: (3)

Sol: $F=-\frac{du}{dr}=\frac{k}{r^3}$

$\frac{mv^2}{a}=\frac{k}{a^3}$

$\Rightarrow KE=\frac{k}{2a^2}$

$PE=\frac{-k}{2a^2}$

TE = 0

5. In a collinear collision, a particle with an initial speed v_o strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

1) $\frac{v_o}{4}$ 2) $\sqrt{2}\,v_o$ 3) $\frac{v_o}{2}$ 4) $\frac{v_o}{\sqrt{2}}$

Key: (2)

Sol: Momentum conservation

$mv_0 = m(v_1 - v_2)$

$v_1 = v_2 = v_0$ ________ (1)

$\frac{1}{2}m(v_1^2+v_2^2)-\frac{1}{2}mv_0^2=\frac{1}{2}\times\frac{1}{2}mv_0^2$

$v_1^2 + v_2^2 = \frac{3}{2}v_0^2$ ________ (2)

$(v_1 - v_2)^2 = v_0^2$

$v_1^2 + v_2^2 - 2v_1v_2 = v_0^2$ ________ (3)

(2) – (3)

$\Rightarrow \quad 2v_1v_2 = \frac{v_0^2}{2}$

$\Rightarrow \quad 4v_1v_2 = v_0^2$

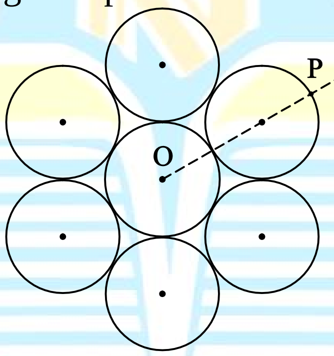
$$(v_1 + v_2) = \sqrt{(v_1 - v_2)^2 + 4v_1v_2}$$

Relative velocity after collision

$$= \sqrt{v_0^2 + v_0^2}$$

$$= \sqrt{2}v_0$$

6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is :



1) $\frac{19}{2}MR^2$ 2) $\frac{55}{2}MR^2$ 3) $\frac{73}{2}MR^2$ 4) $\frac{181}{2}MR^2$

Key: (4)

Sol: Use parallel axis theorem

$I_0 = 6 \times \left[\frac{MR^2}{2}\right] + \left[\frac{MR^2}{2}\right] + 6M(2R)^2$

$= 3MR^2 + \frac{MR^2}{2} + 24MR^2$

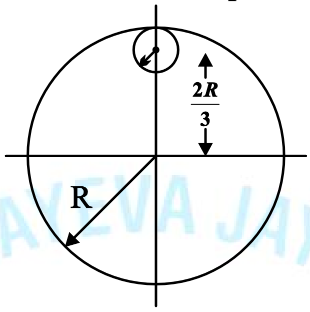
$= \frac{55}{2}MR^2$

$I_P = I_0 + 7M(3R)^2$

$= \frac{55}{2}MR^2 + 63MR^2$

$= \frac{181}{2}MR^2$

7. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :



1) $4MR^2$ 2) $\frac{40}{9}MR^2$ 3) $10MR^2$ 4) $\frac{37}{9}MR^2$

Key: (1)

Sol: $I_{full} = I_{scooped} + I_{remaining}$

$$\frac{9M.R^2}{2} = \left[\frac{MR^2}{2\times 9} + \frac{4MR^2}{9}\right] + I_{rem}$$

$$\frac{9MR^2}{2} = \frac{9}{18}MR^2 + I_{rem}$$

$I_{rem} = 4\ MR^2$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particles is T, then :

1) $T \alpha R^{3/2}$ for any n.

2) $T \alpha R^{\frac{n}{2}+1}$

3) $T \alpha R^{(n+1)/2}$

4) $T \alpha R^{n/2}$

Key: (3)

Sol: $F \propto \frac{1}{R^n}$

$$MR\frac{4\pi^2}{T^2} \propto \frac{1}{R^n}$$

$$\boxed{T \propto R^{\frac{n+1}{2}}}$$

9. A solid sphere of radius r made of a soft material of bulk modules K is surrounded by a liquid in a cylindrical container. A massless piston of area of floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is :

1) $\frac{Ka}{mg}$ 2) $\frac{Ka}{3mg}$ 3) $\frac{mg}{3Ka}$ 4) $\frac{mg}{Ka}$

Key: (3)

Sol: Bulk modulus $=\frac{\Delta p}{\frac{-\Delta v}{v}}$

$$\left[\frac{\Delta v}{v}\right]=\frac{mg}{ak}$$

$$\frac{\Delta r}{r}=\frac{mg}{3ak}$$

10. Two moles of an ideal monoatomic gas occupies a volume V at $27^{o}C$. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

1) (a) 189 K (b) 2.7 kJ
2) (a) 195 K (b) – 2.7 kJ
3) (a) 189 K (b) – 2.7 kJ
4) (a) 195 K (b) 2.7kJ

Key: (3)

Sol: $Tv^{\gamma-1}=\text{const}$

$300\ v^{2/3} = T.\ 2^{2/3}\ v^{2/3}$

$T=\frac{300}{2^{2/3}}=189\,K$

$\Delta u = nc_v\Delta T$

$= -2.7\ KJ$

11. The mass of a hydrogen molecule is $3.32\times10^{-27}kg$. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area $2cm^2$ at an angle of 45^{o} to the normal, and rebound elastically with a speed of $10^3 m/s$, then the pressure on the wall is nearly :

1) $2.35\times10^3 N/m^2$ 2) $4.70\times10^3 N/m^2$ 3) $2.35\times10^2 N/m^2$ 4) $4.70\times10^2 N/m^2$

Key: (1)

Sol: $P=\frac{F}{A}=\frac{\Delta(mv_1)}{\Delta t}=\frac{(2mv_1)}{A}(n/t)$

$$\frac{2\times3.32\times10^{-27}\times\left(1000\sqrt{2}\right)}{2\times10^{-4}}\left(10^{23}\right)$$

$=2.35\times10^3 N/m^2$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}/\text{sec}$. What is the force constant of the bonds connecting one atom with the order ? (Mole wt. of silver = 108 and Avagadro number = $6.02\times10^{23}\,gm\ mole^{-1}$)

1) 6.4 N /m 2) 7.1 N/m 3) 2.2 N/m 4) 5.5 N/m

Key: (2)

Sol: $w=\sqrt{\frac{k}{m}}$

$$2\times\pi\times10^{12}=\sqrt{\frac{\frac{k}{108}}{6\times10^{23}}}$$

k nearly 7.1 N/m

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7\times10^{3}kg/m^{3}$ and its Young's modulus is $9.27\times10^{10}Pa$. What will be the fundamental frequency of the longitudinal vibrations ?

1) 5 kHz 2) 2.5 kHz 3) 10 kHz 4) 7.5 kHz

Key: (1)

Sol: $l=\frac{\lambda}{2}\Rightarrow\lambda=2l$

$$f=\frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

$$=\frac{1}{2\times60/100}\sqrt{\frac{9.27\times10^{10}}{2.7\times10^{3}}}\approx5\,kHz$$

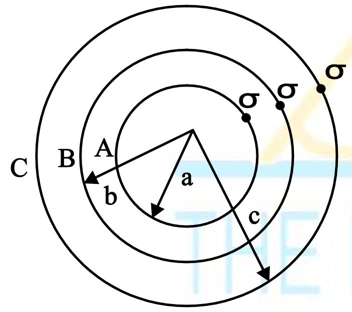
14. Three concentric metal shells A,B and C of respective radii a,b and c (a < b < c) have surface charge densities $+\sigma,-\sigma$ and $+\sigma$ respectively. The potential of shell B is :

1) $\frac{\sigma}{\in_o}\left[\frac{a^2-b^2}{a}+c\right]$ 2) $\frac{\sigma}{\in_o}\left[\frac{a^2-b^2}{b}+c\right]$ 3) $\frac{\sigma}{\in_o}\left[\frac{b^2-c^2}{b}+a\right]$ 4) $\frac{\sigma}{\in_o}\left[\frac{b^2-c^2}{c}+a\right]$

Key: (2)

Sol: $V_B=\frac{1}{4\pi\in_o}\left[\frac{\sigma4\pi a^2}{b}-\frac{\sigma4\pi b^2}{b}+\frac{\sigma4\pi c^2}{c}\right]$

$$V_B=\frac{\sigma}{\in_o}\left[\frac{a^2-b^2}{b}+c\right]$$

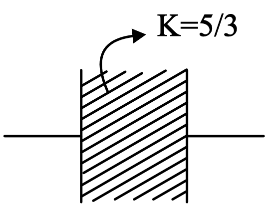


15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K=\frac{5}{3}$ is inserted between the plates, the magnitude of the included charge will be :

1) 1.2 nC 2) 0.3 nC 3) 2.4 nC 4) 0.9 nC

Key: (1)

Sol: $q'=q\left(1-\frac{1}{k}\right)$



$= k\,cv\left(1-\frac{1}{k}\right)$

$= \frac{5}{3}\times 90\times 20\left(1-\frac{3}{5}\right)=1.2\,nc$

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by
e = 100 sin 30 t
$i = 20\sin\left(30t-\frac{\pi}{4}\right)$
In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively :
1) 50, 10 2) $\frac{1000}{\sqrt{2}},10$ 3) $\frac{50}{\sqrt{2}},0$ 4) 50, 0

Key: (2)

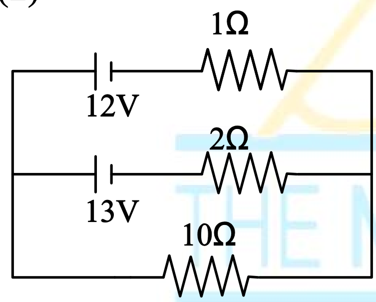
Sol: $p_{avg} = i_{rms}\,v_{rms}\cos\phi$

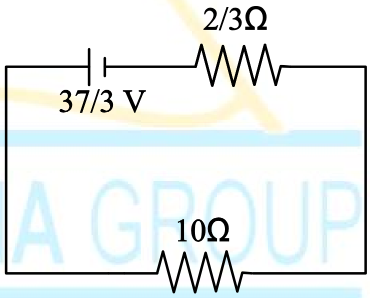
$= \frac{20}{\sqrt{2}}.\frac{100}{\sqrt{2}}\cos\left(\frac{\pi}{4}\right)=\frac{1000}{\sqrt{2}}$

17. Two batteries with e.m.f. 12V and 13V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between :
1) 11.6 V and 11.7 V 2) 11.5 V and 11.6 V 3) 11.4 V and 11.5 V 4) 11.7 V and 11.8V

Key: (2)

Sol:





$E_{eq} = \frac{\frac{\epsilon_1}{r_1}+\frac{\epsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\frac{12}{1}+6.5}{1+\frac{1}{2}} = \frac{37}{3}$

$i = \frac{V}{R} = \frac{37/3}{32/3} = \frac{37}{32}$

$V = i\times R = \frac{370}{32} = 11.56V$

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is :

1) $r_e > r_p = r_\alpha$ 2) $r_e < r_p = r_\alpha$ 3) $r_e < r_p < r_\alpha$ 4) $r_e < r_\alpha < r_p$

Key: (2)

Sol: $r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB} \alpha \sqrt{\frac{k}{q^2}}$

$r_c < r_p = r_\alpha$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is :

1) 2 2) $\sqrt{3}$ 3) $\sqrt{2}$ 4) $\frac{1}{\sqrt{2}}$

Key: (3)

Sol: $m = i\lambda a^2$ $m' = 2m = 2i\pi a^2 = i\pi a'^2$

$a' = \sqrt{2}a$

$$\frac{B_1}{B_2} = \frac{\frac{\mu_o i}{2a}}{\frac{\mu_o i}{2a\sqrt{2}}} = \sqrt{2}$$

20. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_o = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :

1) $\frac{\omega_o L}{R}$ 2) $\frac{\omega_o R}{L}$ 3) $\frac{R}{(\omega_o C)}$ 4) $\frac{CR}{\omega_o}$

Key: (1)

Sol: Direct formula, $Q = \frac{w_o L}{R}$

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z - ct)\right]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non – magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct ?

1) $\frac{\epsilon_{r1}}{\epsilon_{r_2}} = 4$ 2) $\frac{\epsilon_{r1}}{\epsilon_{r_2}} = 2$ 3) $\frac{\epsilon_{r1}}{\epsilon_{r_2}} = \frac{1}{4}$ 4) $\frac{\epsilon_{r1}}{\epsilon_{r_2}} = \frac{1}{2}$

Key: (3)

Sol: $\frac{c_1}{c_2} = 2$

$$c = \frac{1}{\sqrt{\mu_0 \in_0}}$$

$$\in \alpha \frac{1}{c^2}$$

$$\frac{\in_1}{\in_2} = \left(\frac{c_2}{c_1}\right)^2 = \frac{1}{4}$$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{1}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polarizer A and C is :

1) 0^o 2) 30^o 3) 45^o 4) 60^o

Key: (3)

Sol: $I = \frac{I_0}{2}\cos^2\phi$

$I = \frac{I_0}{2}\cos^2\phi_1 \cos^2\phi_2$

$\frac{I_0}{8} = \frac{I_0}{2}\cos^4\phi$

$\theta_1 = \theta_2$

$\cos\phi = \frac{1}{\sqrt{2}}$

$\phi = 45^0$

23. The angular width of the central maximum in a single slit diffraction pattern is 60^0. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

1) 25 μm. 2) 50 μm. 3) 75 μm. 4) 100 μm.

Key: (1)

Sol: $d\sin 30 = \lambda$

$\lambda = 0.5\times10^{-6} m$

$\beta = \lambda\frac{D}{d} = 0.5\times10^{-6}\times\frac{50}{1}$

$= 25\mu m$

24. A electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the nth state and the ground state respectively. Let $\wedge_n$ be the wavelength of the emitted photon in the transition from then nth state to the ground state. For large n, (A,B are constants)

1) $\wedge_n \approx A + \frac{B}{\lambda_n^2}$ 2) $\wedge_n \approx A + B\lambda_n$ 3) $\wedge_n^2 \approx A + B\lambda_n^2$ 4) $\wedge_n^2 \approx \lambda$

Key: (1)

Sol: Debroglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$ $k = 13.6\frac{z^2}{n^2}$

$\therefore \lambda \alpha n$

$\frac{1}{\Lambda} = R\left(\frac{1}{1} - \frac{1}{n^2}\right)$

$\Lambda = \frac{1}{R}\left(1 - \frac{1}{n^2}\right)^{-1} = \frac{1}{R}\left(1 + \frac{1}{n^2}\right) = A + \frac{B}{\lambda^2}$

25. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is:

1) $25\nu_L$ 2) $16\nu_L$ 3) $\nu_L/16$ 4) $\nu_L/25$

Key: (4)

Sol: Series limit for Lyman series, $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$

$f = \frac{v}{\lambda}$ $f = \frac{C}{\lambda} = CR = \beta\nu_L$

Series limit for pfund series, $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{R}{25}$

$f = \frac{C}{\lambda} = \frac{CR}{25} = \frac{\nu_L}{25}$

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively :

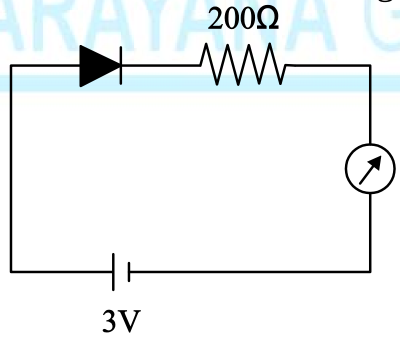
1) (·89, ·28) 2) (·28, ·89) 3) (0, 0) 4) (0, 1)

Key: (1)

Sol: $p_d = \frac{4\times1\times2}{(1+2)^2} = \frac{8}{9} = 0.89$

$p_c = \frac{4\times1\times12}{13^2} = \frac{48}{169} = 0.28$

27. The reading of the ammeter for a silicon diode in the given circuit is:



1) 0 2) 15mA 3) 11.5mA 4) 13.5 mA

Key: (3)

Sol: Drop across 200Ω is 2.3 volt since 0.7 volt is Barrier voltage for diode.

$1^0 = \frac{2.3}{200} = 11.5\,mA$

28. A telephonic communication service is working at carrier frequency of 10GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

1) 2×10^3 2) 2×10^4 3) 2×10^5 4) 2×10^6

Key: (3)

Sol: $= \frac{1\times10^6\,kHz}{5\,kHz} = 2\times10^5$

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, a balance is found when the cell is connected across 40cm of the wire. Find the internal resistance of the cell.

1) 1Ω 2) 1.5Ω 3) 2Ω 4) 2.5Ω

Key: (2)

Sol: $r = R\left[\frac{l_1 - l_2}{l_2}\right]$

$= 5\left[\frac{52-40}{40}\right] = \frac{5\times12}{40} = \frac{60}{40} = 1.5\Omega$

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10cm. The resistance of their series combination is $1k\Omega$. How much was the resistance on the left slot before interchanging the resistances ?

1) 990 Ω 2) 505 Ω 3) 550 Ω 4) 910 Ω

Key: (3)

Sol: $\frac{R_L}{R_R} = \frac{l}{100-l}$

$\frac{R_R}{R_L} = \frac{l-10}{110-l}$

$\frac{R_L}{R_R}\times\frac{R_R}{R_L} = 1$

$\Rightarrow l = 55cm$

$\Rightarrow \frac{R_L}{R_R} = \frac{55}{45} = \frac{11}{9}$

$R_R = \frac{9}{11}R_L$

$R_R + R_L = 1000\Omega$

$\left[\frac{9}{11}+1\right]R_L = 1000$

$R_L = \frac{1000}{20}\times11$

$R_L = 550\Omega$

CHEMISTRY

31. The ratio of mass percent C and H of an organic compound $(C_X H_Y O_Z)$ is 6 : 1. If one molecule of the above compound $(C_X H_Y O_Z)$ contains half as much oxygen as required to burn one molecule of compound $C_X H_Y$ completely to CO_2 and H_2O. The empirical formula of compound $C_X H_Y O_Z$ is :

1) $C_3H_6O_3$ 2) C_2H_4O 3) $C_3H_4O_2$ 4) $C_2H_4O_3$

Key: (4)

Sol: $C_xH_y + \left(X + \frac{Y}{4}\right)O_2 \rightarrow XCO_2 + \frac{Y}{2}H_2O$

According to given statement

$Z = \frac{1}{2}\left(2x + \frac{y}{2}\right)$

$X = 2$; $Y = 4$; $Z = 3$

32. Which type of 'defect' has the presence of cations in the interstitial sites?

1) Schottky defect 2) Vacancy defect
3) Frenkel defect 4) Metal deficiency defect

Key: (3)

Sol: Frenkel defect is due to presence of cation in interstitial site

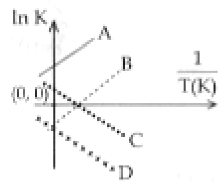
33. According to molecular orbital theory, which of the following will not be a viable molecule?

A) He_2^{2+} B) He_2^{+} C) He_2^{-} D) He_2^{2-}

Key: (4)

Sol: H_2^{2-} is not a viable molecule according to MOT

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



1) A and B 2) B and C 3) C and D 4) A and D

Key: (1)

Sol: A and B have positive slopes for which ΔH is negative

35. The combustion of benzene (1) gives $CO_2(g)$ and $H_2O(1)$. Given that heat of combustion of benzene at constant volume is $-3263.9\ kJ\ mol^{-1}$ at 25^0C; heat of combustion (in kJ mole^{-1}) of benzene at constant pressure will be : $(R = 8.314\ JK^{-1}\ mol^{-1})$

1) 4152.6 2) -452.46 C) 3260 D) -3267.6

Key: (4)

Sol: $C_6H_6 + \frac{15}{2}O_{2(g)} \rightarrow 6CO_{2(g)} + 3H_2O_{(l)}$

$6-\frac{15}{2}=\frac{-3}{2}$

$\Delta H=\Delta E+\Delta nRT$

$=-3263.9-\frac{3}{2}\times 8.314\times 10^{-3}\times 298$

$=-3263.9-3.716$

$=-3267.676$

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

1) $\left[Co\left(H_2O\right)\right]Cl_3$
2) $\left[Co\left(H_2O\right)_5Cl\right]Cl_2.H_2O$
3) $\left[Co\left(H_2O\right)_4Cl_2\right]Cl.2H_2O$
4) $\left[Co\left(H_2O\right)_3Cl_3\right].3H_2O$

Key: (4)

Sol: Freezing point is inversely proportional to number of particles

$T_f\propto\frac{1}{i}$

37. An aqueous solution contains $0.10\,M\ H_2S$ and $0.20\,M\ HCl$. If the equilibrium constant for lthe formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

1) 5×10^{-8}
2) 3×10^{-20}
3) 6×10^{-21}
4) 5×10^{-19}

Key: (2)

Sol: $H_2S \rightleftharpoons HS^-+H^+\quad Ka_1=1\times 10^{-7}$

$HS^- \rightleftharpoons H^++S^{-2}\ Ka_2=1.2\times 10^{-13}$

$H_2S \rightleftharpoons 2H^++S^{-2}$

$1\times 10^{-7}\times 1.2\times 10^{-13}=\frac{\left(H^+\right)^2\left(S^{-2}\right)}{\left(H_2S\right)}$

$=\frac{0.2\times 0.2\times\left(S^{-2}\right)}{0.1}$

$\left(S^{-2}\right)=\frac{10^{-7}\times 1.2\times 10^{-13}\times 10^{-1}}{2\times 2\times 10^{-2}}$

$\frac{1.2}{4}\times 10^{-19}$

$=0.3\times 10^{-19}$

$=3\times 10^{-10}$

$10^{-7}\times 10^{-13}\times 1.2=\frac{\left[H^+\right]^2\left[S^{-2}\right]}{\left[H_2S\right]}$

38. An aqueous solution contains unknown concentration of Ba^{2+}. When 50 mL of a 1M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is $500\,mL$. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?

1) $5\times 10^{-9}M$
2) $2\times 10^{-9}M$
C) $1.1\times 10^{-9}M$
D) $1.0\times 10^{-10}M$

Key: (3)

Sol: $BaSO_4 \rightleftharpoons Ba^{+2} + SO_4^{-2}$

$\underset{(0.05)}{Na_2SO_4} \rightarrow 2Na^{+} + \underset{\left(\frac{0.05}{0.5}\right)=10^{-1}}{SO_4^{-2}}$

$KS_P = \left[Ba^{+2}\right]\left[SO_4^{-2}\right]$

$10^{-10} = \left[Ba^{+2}\right]\left(10^{-1}\right)$

$\therefore \left[Ba^{+2}\right] = \left[10^{-9}\right]$

$V_1 = 500 \quad V_2 = 450$

$M_1 = 10^{-9} \quad M_2 = ?$

$500 \times 10^{-9} = 450 \times M_2$

$M_2 = \frac{500}{450} \times 10^{-9} = 1.1 \times 10^{-9}$

39. At 518^0C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

1) 2 B) 3 C) 1 D)0

Key: (1)

Sol: $r_1 = k\left(p_1\right)^n$

$r_2 = k\left(p_2\right)^n \Rightarrow \frac{r_1}{r_2} = \left(\frac{p_1}{p_2}\right)^n$

$\frac{1}{0.5} = \left(\frac{344.85}{241.7}\right)^n$

$2 = \left(1.42\right)^n$

$\therefore n = 2$

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66g of diborane? (Atomic weight of B = 10.8 u)

1) 6.4 hours 2) 0.8 hours 3) 3.2hours 4) 1.6hours

Key: (3)

Sol: $B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$

$\left(\frac{27.66gm}{27.6}\right) = 1\,mole$

$\left(1\,mole\right)$ *of* B_2H_6 *requires* 3 *moles of* O_2 required

$2H_2O \xrightarrow{4F} \underset{(1\,mole)}{O_2} + 2H_2$

$1 mole\ of\ O_2 - \left(4F\right)$

$3\,mole - \ ? \quad = 12\,F$

$12 \times 96500 = 100 \times t$

$t = 11,580\ \sec$

$t = 3.2\ hr$

41. The recommended concentration of fluoride ion in drinking water is up to 1ppm as fluoride ion is required to make teeth enamel harder by converting $\left[3Ca_3(PO_4)_2.Ca(OH)_2\right]$ to :

1) $[CaF_2]$
2) $\left[3(CaF_2).Ca(OH)_2\right]$
3) $\left[3Ca_3(PO_4)_2.CaF_2\right]$
4) $\left[3\{Ca(OH_2)_2\}.CaF_2\right]$

Key: (3)

Sol: $Ca_3(PO_4)_2.Ca(OH)_2 \rightarrow Ca_3(PO_4).CaF_2$

42. Which of the following compounds contain(s) no covalent bond(s)?
$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

1) KCl, B_2H_6, PH_2
2) KCl, H_2SO_4
3) KCl
4) KCl, B_2H_6

Key: (3)

Sol: only KCl has ionic bond

43. Which of the following are Lewis acids?

1) PH_3 and BCl_3
2) $AlCl_3$ and $SiCl_4$
3) PH_3 and $SiCl_4$
4) BCl_3 and $AlCl_3$

Key: (2 or 4)

Sol: $AlCl_3, BCl_3$ are Electron deficient

44. Total number lone pair of electrons in I_3^- ion is :

1) 3
2) 6
3) 9
4) 12

Key: (3) 0

Sol: $:\ddot{\underset{..}{I}} - \ddot{\underset{..}{I}}:^{(-)} - \ddot{\underset{..}{I}}: \rightarrow$ has 9 lone pairs

45. Which of the following salts is the most basic in aqueous solution?

1) $Al(CN)_3$
2) CH_3COOK
3) $FeCl_3$
4) $Pb(CH_3COO)_2$

Key: (2)

Sol: $CH_3COO^{(-)}$ undergoes anionic hydrolysis and among CH_3COOK has higher pH

46. Hydrogen peroxide oxidizes $\left[Fe(CN)_6\right]^{4-}$ to $\left[Fe(CN)_6\right]^{3-}$ in acidic medium but reduces $\left[Fe(CN)_6\right]^{3-}$ to $\left[Fe(CN)_6\right]^{4-}$ in alkaline medium. The other products formed are, respectively :

1) $(H_2O + O_2)$ and H_2O
2) $(H_2O + O_2)$ and $(H_2O + OH^-)$
3) H_2O and $(H_2O + O_2)$
4) H_2O and $(H_2O + OH^-)$

Key: (3)

Sol: H_2O_2 oxidizing properties

$H_2O_2 + \left[Fe(CN)_6\right]^{4-} \rightarrow \left[Fe(CN)_6\right]^{3-} + H_2O$

H_2O_2 Reducing properties

$H_2O_2 + \left[Fe(CN)_6\right]^{-3} \rightarrow \left[Fe(CN)_6\right]^{4-} + H_2O + O_2$

47. The oxidation states of Cr in $\left[Cr\left(H_2O\right)_6\right]Cl_3, \left[Cr\left(C_6H_6\right)_2\right]$, and $K_2\left[Cr\left(CN\right)_2\left(O\right)_2\left(O\right)_2\left(NH_3\right)\right]$ respectively are :

1) $+3,+4$ and $+6$ 2) $+3,+2$ and $+4$ 3) $+3,0$ and $+6$ 4) $+3,0$ and $+4$

Key: (4)

Sol: $\left[Cr\left(H_2O\right)_6\right]Cl_3$

$x+0-3=0$

$x=+3$

$\left(CrCC_6H_6\right)_2$

$x+0=0$

$x=0$

$k_2\left[G\left(CN\right)_2\left(O\right)_2\left(O\right)_2\left(NH_3\right)\right]$

$+2+x-2-4+0+0=0$

$x=+4$

48. The compound that does not produce nitrogen gas by the thermal decomposition is :

1) $Ba\left(N_3\right)_2$ 2) $\left(NH_4\right)_2Cr_2O_7$ 3) NH_4NO_2 4) $\left(NH_4\right)_2SO_4$

Key: (4)

Sol: $Ba\left(NH_3\right)_2 \rightarrow Ba+N_2$

$\left(NH_4\right)_2Cr_2O_7 \rightarrow N_2+CrO_3$

$NH_4NO_2 \rightarrow N_2+H_2O$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

1) Zn 2) Ca 3) Al 4)Fe

Key: (3)

Sol: $Al+NaOH \rightarrow Al\left(OH\right)_3\downarrow$

$Al\left(OH\right)_3 \xrightarrow{NaOH\ excess} NaAlO_2$ Soluble

$Al\left(OH\right)_3 \xrightarrow{\Delta} Al_2O_3$

50. Consider the following reaction statements:

$\left[Co\left(NH_3\right)_4Br_2\right]^+ + Br^- \rightarrow \left[Co\left(NH_3\right)_3Br_3\right] + NH_3$

I) Two isomers are produced if the reactant complex ion is a cis-isomer

II) Two isomers are produced if the reactant complex ion is a transisomer.

III) Only one isomer is produced if the reactant complex ion is a transisomer

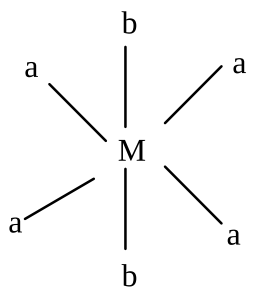
IV) Only one isomer is produced if the reactant complex ion is a cis-isomer.

The correct statements are :

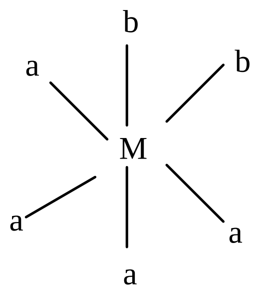
1) (I) and (II) 2) (I) and (III) 3) (III) and (IV)4) (II) and (IV)

Key: (2)

Sol:



Trans



Due to presence of phase of symmetry, III & IV are correct

51. Glucose on prolonged heating with HI gives :

1) n-Hexan 2) 1-Hexene 3) Hexanoic acid 4) 6-iodohexanal

Key: (1)

Sol: $Glu\cos e \xrightarrow[\substack{HI \\ \text{Prolongad} \\ \text{Healing}}]{\text{Red } P_4} n - Hexan$

52. The trans-alkenes are formed by the reduction of alkynes with :

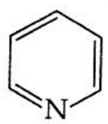
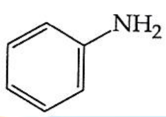
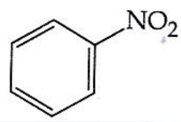
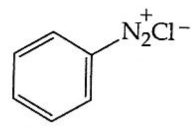
1) $H_2 - Pd / C, BaSO_4$ 2) $NaBH_4$

3) $Na / liq.NH_3$ 4) $Sn - HCl$

Key: (3)

Sol: Birch reduction gives trans alkene with alkyne

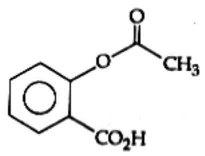
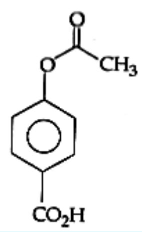
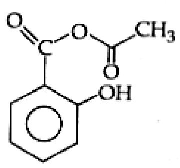
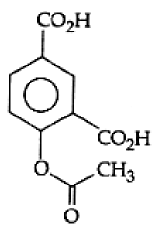
53. Which of the following compounds will be suitable for Kjeldhal's method for nitrogen estimation?

(1)  (2)  (3)  (4) 

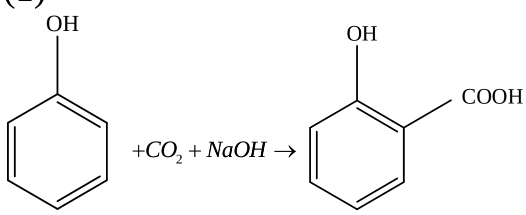
Key: (2)

Sol: Nitrogen in the ring & $-NO_2$ groups & $-N_2^+Cl^-$ fails kjeldah's test

54. Phenol on treatement with CO_2 in the presence of NaOH followed by acidification product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produceds :

(1)  (2)  (3)  (4) 

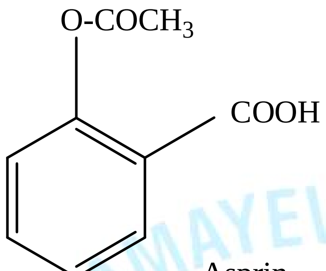
Key: (1)

Sol: 

Alicylicociol (x)

(Major)

$\downarrow (CH_3CO)_2O / H^+$



Asprin

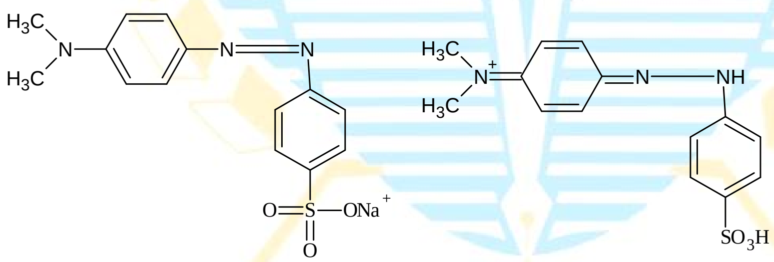
55. An alkali is titrated agains an acid with methyl orange an indicator, which of the following is a correct combination ?

	Base	Acid	End point
1)	Weak	Strong	Colourless to pink
2)	Strong	Strong	Pinkish red to yellow
3)	Weak	Strong	Yellow to pinkish red
4)	Strong	Strong	Pink to colourless

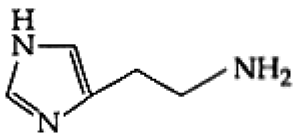
Key: (3)

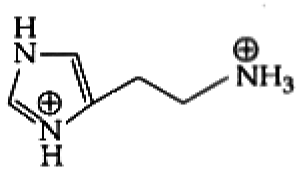
Sol: For methyl orange

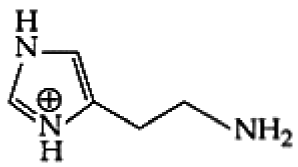
$Strong\ acid + Base \Rightarrow \underset{}{\overset{(pH>7)}{Yellow}}$ to $\overset{(pH<7)}{Pink}$

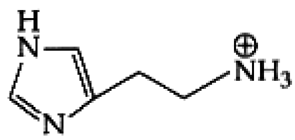


56. The predominate form of histamine prexent in human blood is (pKa, Histidine = 6.0)

(1) 

(2) 

(3) 

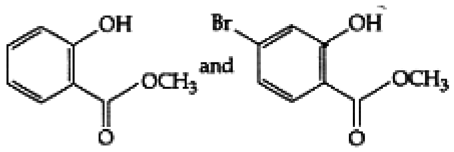
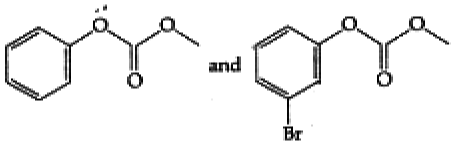
(4) 

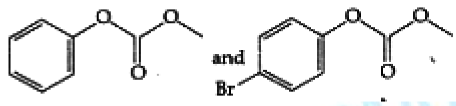
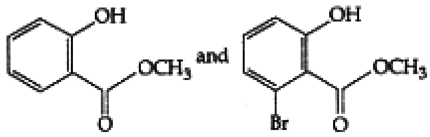
Key: (4)

Sol: p^H of blood = 7.35

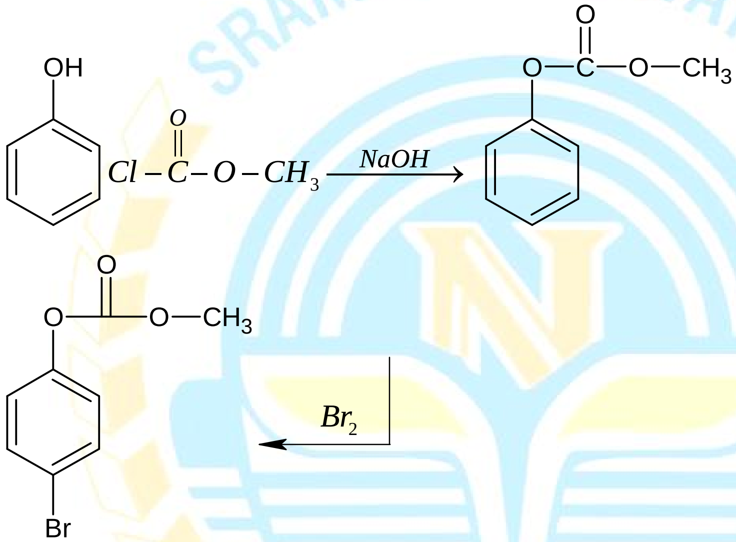
Above the Iso-electric point medium, Hence, Ink plasma exist as monocationic form

57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

(1) 
(2) 

(3) 
(4) 

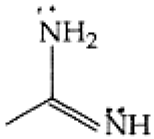
Key: (3)

Sol:


58. The increasing order of basicity of the following compounds is :

(a) 

(b) 

(c) 

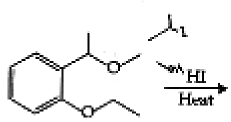
(d) 

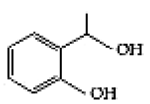
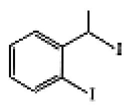
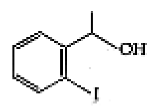
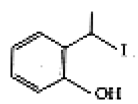
1) a < b < c < d 2) b < a < c < d 3) b < a < d < c 4) d < b < a < c

Key: (3)

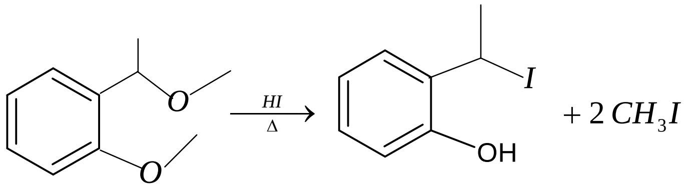
Sol: c > d > a > b

59. The major product formed in the following reaction is :

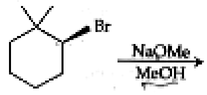


(1) 
(2) 
(3) 
(4) 

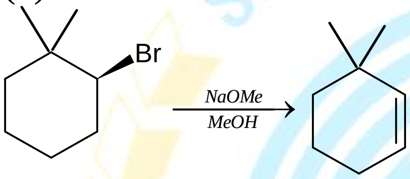
Key: (4)

Sol: 

60. The major product of the following reaction is :



Key: (2)

Sol: 

MATHEMATICS

61. Two sets A and B are as under: $A=\{(a,b)\in R\times R:|a-5|<1 \text{ and } |b-5|<1\}$;

$B=\{(a,b)\in R\times R:4(a-6)^2+9(b-5)^2\le 36\}$ Then :

(1) $B\subset A$ (2) $A\subset B$

(3) $A\cap B=\phi$ (an empty set) (4) neither $A\subset B$ nor $B\subset A$

Key: (2)

Sol: $|a-5|<1$ *and* $|b-5|<1$

$-1<a-5<1$ *and* $-1<b-5<1$

$A=\{4<a<6$ *and* $4<b<6\}$

$4(a-6)^2+9(b-5)^2\le 36$

$\frac{(a-6)^2}{9}+\frac{(b-5)^2}{4}\le 1$

A C B

62. Let $S=\{x\in R:x\ge 0$ and $2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S:

(1) is an empty set. (2) contains exactly one element

(3) contains exactly two element (4) contains exactly four element

Key: (3)

Sol: $\sqrt{n}=t$ $\quad x=t^2$

$2|t-3|+t^2-bt+6=0$

$t\ge 3$ $\quad 2t-6+t^2-6t+6=0$

$t^2-4t=0$ $\quad t=0$ & $t=4$ $\quad$ x = 16

$t<3$ $\quad 6-2t+t^2-6t+6=0$

$t^2-8t+12=0$

$t=\frac{8\pm\sqrt{64-48}}{2}$

$=\frac{8\pm 4}{2}=6,2$

X = 4 ; x = 4 & 16

63. If α, β ε C, are the distinct roots, of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to:

(1) – 1 (2) 0 (3) 1 (4) 2

Key: (3)

Sol: $x^2 - x + 1 = 0$

$x = -w, -w^2$

$\alpha^{101} + \beta^{107} = (-w)^{(0)} + (w^2)^{107}$

$= -\left(w^{(0)} + w^{214}\right)$

$= -\left(w^2 + w\right) = 1$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to:

(1) (–4, –5) (2) (–4, (3) (3) (–4, 5) ((4) (4, 5)

Key: (3)

Sol: $R_1 \to R_1 + R_2 + R_3$

$$(5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 + R_3 - R_1$

$$(5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -(x+4) & 0 \\ 0 & 0 & -(x+4) \end{vmatrix}$$

$(5x-4)(x+4)^2$

$A = -4 \quad B = 5$

65. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to:

(1) –10 (2) 10 (3) –30 (4) 30

Key: (2)

Sol: $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\{-3k + 8\} - k\{-9 + 4\} + 3\{12 - 2k\} = 0$

$8-3k+5k+36-6k=0$

$4k=44$

$k=11$

$x+11y=-3p$ where t = p

$3x+11y=2p$

$2x=5p \Rightarrow x=\frac{5p}{2}$

$11y=\frac{-11p}{2} \Rightarrow y=\frac{-p}{2}$

$$\frac{xz}{y^2}=\frac{\left(\frac{5p}{2}\right)\times p}{\left(-\frac{p}{2}\right)^2}=\frac{5p}{2}\times\frac{4}{p^2}=10$$

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 1000 (2) less than 500

(3) at least 500 but less than 750 (4) at least 750 but less than 1000

Key: (1)

Sol: $6C_4.3C_1.4! = 1080$

67. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, (x > (1) is :

(1) –1 (2) 0 (3) 1 (4) 2

Key: (4)

Sol: $\left(x+\sqrt{x^3-1}\right)^5=x^5+5c_1x^4\sqrt{x^3-1}+5c_2x^3\left(x^3-1\right)+5c_3x^2\left(\sqrt{x^3-1}\right)^3+5c_4x\left(x^3-1\right)^2+5c_5\left(\sqrt{x^3-1}\right)^5$

$\left(x-\sqrt{x^3-1}\right)^5=x^5-5c_1\ x^4\sqrt{x^3-1}+5c_2x^3\left(x^3-1\right)-5c_3x^2\left(\sqrt{x^3-1}\right)^3+5c_4x\left(x^3-1\right)^2+5c_5\left(\sqrt{x^3-1}\right)^5$

Adding we have

$2x^5+20\,x^3\left(x^3-1\right)+10x\left\{x^6-2x^3+1\right\}$

$2x^5+20x^6-20x^3+10x^7-20x^4+10x$

$10+2-20+10=2$

68. Let a_1, a_2, a_3, …. A_{49} be in A.P. such that $\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9 + a_{43} = 66$. If $a_1^2+a_2^2+.....+a_{17}^2=140\,m$, then m is equal to:

(1) 66 (2) 68 (3) 34 (4) 33

Key: (3)

Sol: $a_1 + a_5 + a_9 + a_{13} + ... + a_{49} = 416$

$a_1 + a_1 + 4d + a_1 + 8d + ... + a_1 + 48d = 416$

$12a_1 + 4d\{1 + 2 + 3 + ... + 12\} = 416$

$12a_1 + 4d_1 \frac{(12)(13)}{2} = 416 \Rightarrow 12a_1 + 312d = 416$

$a_1 + 26d_1 = \frac{416}{12} \qquad \Rightarrow 12a_1 + 300d = 792$

$a_1 + 8d + a_1 + 42d = 66$

$12d = -376$

$d = -376$

$a_1 = \frac{139}{4}$

$a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$

$17a_1 + 2a_1 d\{1 + 2 + 3 + ... + 16\} + d^2\{1 + 2^2 + ... + 16^2\} = 140m$

By simplify we get $m = 34$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$
If $B - 2A = 100\lambda$, then λ is equal to:

(1) 232 (2) 248 (3) 464 (4) 496

Key: (2)

Sol: $A = (1^2 + 2^2 + ... + 20^2) + (2^2 + 4^2 + 6^2 + ... + 20^2)$

$B = \frac{40 \times 41 \times 8}{6} + \frac{4 \times 20 \times 21 \times 41}{6} - \frac{2 \times 200 \times 2(\times 4)}{6}$

$A - 2B = 248$

70. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x \to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

(1) is equal to 0. (2) is equal to 15.

(3) is equal to 120. (4) does not exist (in R).

Key: (3)

Sol: $\underset{x\to 0}{Lt}\, x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}+\frac{2}{x}-\left\{\frac{2}{x}\right\}+....\right)$

$x\left(\frac{1+2+3+....+15}{x}-\left\{\frac{1}{x}\right\}-\left\{\frac{2}{x}\right\}-.....-\left\{\frac{15}{x}\right\}\right)$

$=120$

71. Let $S=\{t\in R: f(x)=|| x-\pi.(e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to :

(1) (ϕ an empty set) (2) {0} (3) {π} (4) {0, π}

Key: (1)

Sol: Check at $x=0$ *and* π

$\left.\begin{matrix} L.H.D=0 \\ R.H.D=0 \end{matrix}\right\}$ *Both at* $x=0\ \&\ \pi$

differentiable at $x=0\ \&\ \pi$

72. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

(1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

Key: (4)

Sol: $y^2=6x\ \&\ 9x^2+16b^2=16$

$\left(\frac{dy}{dx}\right)_1\times\left(\frac{dy}{dx}\right)_2=-1$

$b=\frac{9}{2}$

73. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}$, $x\in R-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$ then the local minimum value of h(x) is:

(1) 3 (2) –3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

Key: (4)

Sol: $h(x)=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}$

$x-\frac{1}{x}=t$

$h(t)=\frac{t^2+2}{t}=t+\frac{2}{t}$

At $x=\sqrt{2}$

$\therefore$ local min $=2\sqrt{2}$

74. The integral $\int \frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2+\cos^5 x\right)}dx$ is equal to:

(1) $\frac{1}{3\left(1+\tan^3 x\right)}+C$ (2) $\frac{-1}{3\left(1+\tan^3 x\right)}+C$ (3) $\frac{1}{1+\cot^3 x}+C$ (4) $\frac{-1}{1+\cot^3 x}+C$

(where C is a constant of integration)

Key: (2)

Sol: $\int \frac{\sin^2 x\cos^2 x\quad dx}{\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x}$

$\int \frac{\tan^2 x\sec^4 x\sec^2 x\, dx}{\left(\tan^5 x+\tan^2 x+\tan^3 x+1\right)^2}$

$\int \frac{t^2 dt}{\left(t^3+1\right)^2}=\frac{-1}{3\left(1+\tan^3 x\right)}+c$

75. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is:

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$ (3) 4π (4) $\frac{\pi}{4}$

Key: (4)

Sol: $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^{-x}}dx$

$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

$I=\frac{\pi}{4}$

76. Let g(x) = cos x^2, $f(x)=\sqrt{x}$ and α, β ($\alpha<\beta$) be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve y = (gof)(x) and the lines $x=\alpha$, $x=\beta$ and y = 0, is :

(1) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (2) $\frac{1}{2}\left(\sqrt{3}+1\right)$ (3) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (4) $\frac{1}{2}\left(\sqrt{2}-1\right)$

Key: (1)

Sol: $gof=\cos x$

$$18x^2 - 9\pi x + \pi^2 = 0$$

$$\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$$

$$\int_{\pi/6}^{\pi/3} \cos x\, dx = \frac{\sqrt{3}-1}{2}$$

77. Let y = y(x) b the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{4}{9\sqrt{3}}\pi^2$ (2) $\frac{-8}{9\sqrt{3}}\pi^2$ (3) $\frac{-8}{9}\pi^2$ (4) $-\frac{4}{9}\pi^2$

Key: (3)

Sol: $six\frac{dy}{dx} + y\cos x = 4x$

$$\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$$

$$I = e^{in(\sin x)} = \sin x$$

$$y.\sin x = 2x^2 + c$$

$$y \times \frac{1}{2} = \frac{2\pi^2}{36} - \frac{\pi^2}{2}$$

$$y = \frac{-8\pi^2}{9}$$

78. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) 3x + 2y = 6 (2) 2x + 3y = xy (3) 3x + 2y = xy (4) 3x + 2y = 6xy

Key: (3)

Sol: $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{2}{a} + \frac{3}{b} = 1$$

$$2b + 3a = ab$$

$$2y + 3x = xy$$

79. Let the orthocenter and centroid of a triangle be A(– 3, 5) and B(3, 5) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $\sqrt{10}$ (2) $2\sqrt{10}$ (3) $3\sqrt{\frac{5}{2}}$ (4) $\frac{3\sqrt{5}}{2}$

Key: (3)

Sol: $\underset{(-3,5)}{O}\ \underset{(3,-3)}{G}\ \underset{(\alpha,\beta)}{C}$

$2\alpha - 3 = 9 \quad \alpha = 6$

$2\beta + 5 = 9 \quad \beta = 2$

$OC = \sqrt{9^2 + 3^2} = 3\sqrt{10}$

$r = \frac{3\sqrt{10}}{2} = \quad 3\sqrt{\frac{5}{2}}$

80. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is:

(1) 195 (2) 185 (3) 85 (4) 95

Key: (4)

Sol: $x = \frac{y+7}{2} - 6$

$2x = y - 5 \Rightarrow 2x - y + 5 = 0$

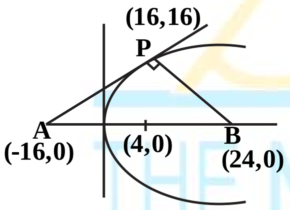
$c(-8,-6) \quad \left|\frac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-c}$

C = 95

81. Tangent and normal are drawn at P(16, 16) on the parabola y2 = 16x, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle$CPB, then a value of tan θ is:

(1) $\frac{1}{2}$ (2) 2 (3) 3 (4) $\frac{4}{3}$

Key: (2)



Sol:

$c\left(\frac{-16+24}{2}, 0\right) \quad c(4,0)$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = 2$

82. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0,3)$ then the area (in sq. units) of ΔPTQ is:

(1) $45\sqrt{5}$ (2) $54\sqrt{3}$ (3) $60\sqrt{3}$ (4) $36\sqrt{5}$

Key: (1)

Sol: $\frac{x^2}{9}-\frac{y^2}{36}=1 \quad y=-12$

$$\frac{x^2}{9}-\frac{12\times 12}{12\times 3}=1$$

$$x^2=5\times 9$$

$$x=3\sqrt{5}$$

$$A=\frac{1}{2}\times 2\times 3\sqrt{5}\times 15=45\sqrt{5}$$

83. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0$, $x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0, 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is:

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Key: (2)

Sol: $y=0$

$$2x+3z-2=0$$

$$2x+2z+2=0$$

$$z=4 \qquad x=-5$$

$$\begin{vmatrix} i & j & k \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = i+j$$

Plane $\Rightarrow (x+5)(-7)+y(7)+(z-4)(-8)=0$

$$distance=\frac{|-35+32|}{\sqrt{162}}$$

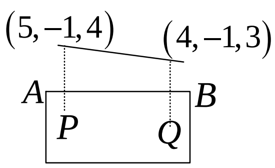
$$=\frac{1}{\sqrt{18}}=\frac{1}{3\sqrt{2}}$$

84. The length of the projection of the line segment joining the points $(5,-1,4)$ and $(4,-1,3)$ on the plane, $x+y+z=7$ is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\sqrt{\frac{2}{3}}$

Key: (4)

Sol:



$AB=\sqrt{1+1}=\sqrt{2}$

$\cos\theta=\frac{1+1}{\sqrt{3}\sqrt{2}}=\frac{2}{\sqrt{3}\sqrt{2}}=\sqrt{\frac{2}{3}}$

$\sin\theta=\frac{PQ}{\sqrt{2}}$

$PQ=\frac{\sqrt{2}}{\sqrt{3}}$

85. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to :

(1) 336 (2) 315 (3) 256 (4) 84

Key: (1)

Sol: $\vec{V}=h\,\vec{a}\times(\vec{a}\times\vec{b})=\lambda\left|\vec{a}(\vec{a}.\vec{b})-\vec{b}|a|^2\right|$

$\vec{V}=\lambda\left|2\vec{a}-14\vec{b}\right|$

$\vec{V}.\vec{b}=24=\lambda\left|2\vec{a}.\vec{b}-14|b|^2\right|$

$\left|\vec{V}\right|^2=\left|14\vec{b}-2\vec{a}\right|^2=336$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{3}{10}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{3}{4}$

Key: (2)

Sol: 4 Red 6Blue

$P(R_2)=P(BR)+P(RR)$

$=\frac{6}{10}\times\frac{4}{12}+\frac{4}{10}\times\frac{6}{12}$

$=\frac{48}{120}=\frac{2}{5}$

87. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1,x_2,......,x_9$ is :

(1) 9 (2) 4 (3) 2 (4) 3

Key: (3)

Sol: $\sum_{i=1}^{9} xi = 9$

$$S.D = \sqrt{\frac{\Sigma(x_i-5)^2}{9} - \left(\frac{\Sigma(xi-j)}{9}\right)^2}$$

$$= \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2} = \sqrt{\frac{36}{9}} = 2$$

88. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to :

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$ (3) $\frac{8}{9}$ (4) $\frac{20}{9}$

Key: (2)

Sol: $8\cos x\left\{\left[\cos^2\frac{\pi}{6}-\sin^2 x\right]-\frac{1}{2}\right\}=1$

$$8\cos x\left\{\left[\frac{-3}{4}-\cos^2 x\right]\right\}=1$$

$$-6\cos x+8\cos^3 x=1$$

$$2\left[4\cos^3 x-3\cos x\right]=1$$

$$\cos 3x=\frac{1}{2}$$

$$x=\frac{2n\pi}{3}\pm\frac{\pi}{9}: n\in z$$

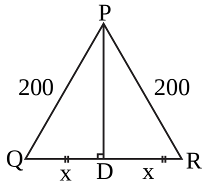
$n=0, n=1$ solution in $[0,\pi]$ $sum=\frac{13\pi}{9}$

89. PQR is a triangular park with $PQ=PR=200m$. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45^0, 30^0$ and 30^0, then the height of the tower (in m) is :

(1) 100 (2) 50 (3) $100\sqrt{3}$ (4) $50\sqrt{2}$

Key: (1)

Sol:



$$\frac{h}{pD}=1$$

$pD = h \quad \frac{h}{x} = \frac{1}{\sqrt{3}} \quad x = \sqrt{3}\ h$

$PD = h = \sqrt{200^2 - x^2} \quad h = 100$

90. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

(1) $\sim p$ (2) p (3) q (4) $\sim q$

Key: (1)

Sol:

P	Q	$\sim P$	pvq	$\sim(pvq)$	$\sim pvq$	$\sim(pvq)v(\sim pvq)$
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T
T	T	F	T	F	F	F

$\sim(pvq)v(\sim p\Lambda q) \cong \sim np$