Rao IIT Academy
Symbol of Excellence and Perfection
JEE | MEDICAL-UG | BOARDS | KVPY | NTSE | OLYMPIADS

Date: 08 Apr 2018

JEE - MAIN
OFFICIAL EXAM - 2018
CODE - C
Solutions

1.

Difficulty : Easy

Topics :

Ionic equilibrium,

$$CH_3COOK + H_2O \rightleftharpoons \underbrace{CH_3COOH}_{\text{weak acid}} + \underbrace{KOH}_{\text{strong base}}$$

2.

Difficulty : Easy

Topics :

PRACTICAL ORGANIC CHEMISTRY,

To estimate $'N'$ by using kjeldhals method, given organic compound must contain $N-H$ bond.
So aniline is ans for given question.

3.

Difficulty : Easy

Topics :

Chemical Bonding,

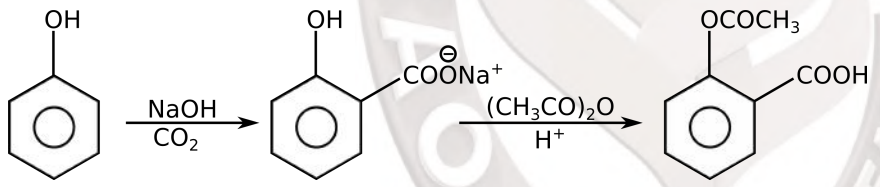
Lewis Acids means molecules having incomplete octet configuration is shown by $BCl_3, AlCl_3$
Electron Rich : pH_3
Octet configuration : $SiCl_4$

4.

Difficulty : Medium

Topics :

Phenols,



5.

Difficulty : Medium

Topics :

Volumetric analysis,

pH range of methly orange is $(3.1-4.5)$
Thus it colour changes from yellow to pinkish red. Initially in basic solution is shows yellow color but as we approaches equivalence/end point is color changes to pinkish red.

6.

Difficulty : Medium

Topics :

Ionic equilibrium,

$$\begin{array}{lllll} H_2S & \rightleftharpoons & H^+ & + & HS^- \\ c-x & & m+x+y & & x-y \end{array} \qquad Ka_1 = 10^{-7}$$

$$[HCl] = 0.2\ M$$

$$[H_2S] = 0.1M$$

$$\begin{array}{lllll} HS^- & \rightleftharpoons & H^+ & + & S^{-2} \\ x-y & & m+x+y & & y \end{array} \qquad Ka_2 = 1.2 \times 10^{-7}$$

$$Ka_1 = \frac{[H^+][HS^-]}{CH_2S} = \frac{(M+x+y)(x-y)}{c-x} = \frac{0.2 \times x}{0.1} \qquad M >>> x >> y$$

$$x = 0.5 \times 10^{-7} \qquad C >> x$$

$$Ka_2 = 1.2 \times 10^{-13} = \frac{[H^+][S^{-2}]}{[HS^-]} = \frac{(m+x+y)(y)}{x-y} = \frac{0.2 \times y}{0.5 \times 10^{-7}}$$

$$y = [S^{-2}] = 3 \times 10^{-20}$$

7.

Difficulty : Medium

Topics :
Thermodynamics,

$$C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$$

$$\Delta u = -3263.9\ kJ/mol$$

$$\Delta H = \Delta u + \Delta ngRT$$

$$= -3263.9 + \frac{(-1.5) \times 8.314 \times 298}{1000}$$

$$\Delta H = -3267.6\ kJ/mol$$

8.

Difficulty : Easy

Topics :

P-block,

$(NH_4)_2SO_4$ does not undergo thermal decomposition.

$$(NH_4)_2Cr_2O_7 \rightarrow N_2 \uparrow + Cr_2O_3 + 4H_2O$$

$$NH_4NO_2 \rightarrow N_2 \uparrow + 2H_2O$$

$$Ba(N_3)_2 \rightarrow Ba + 3N_2 \uparrow$$

9.

Difficulty : Medium

Topics :

Electrochemistry,

$$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$$

mass $27.66\ g$

mole 1

$\therefore$ mole of O_2 react $= 3$

$w = z \times i \times t$

$32 \times 3 = \frac{E}{F} \times i \times t$

$32 \times 3 = \frac{8}{96500} \times 100 \times t$

$t = 3.2\ hrs$

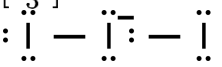
10.

Difficulty : Medium

Topics :

Chemical Bonding,

$[I_3^-]$ radical contains 9 lone pairs in total. $(\cdot\cdot) \longrightarrow$ lone pair.



11.

Difficulty : Medium

Topics :

P-block,

$Al + \underset{\text{excess}}{NaOH} \longrightarrow \underset{(X)}{Al(OH)_3}$ White gelatinous ppt.

$Al + 2NaOH \longrightarrow \underset{(soluble)}{NaAlO_2} + H_2 \uparrow$

$2\underset{X}{Al(OH)} \xrightarrow{\Delta} Al_2O_3 + 3H_2O$

$\downarrow$ absorbent in chromatography

12.

Difficulty : Easy

Topics :

Chemical Bonding,

Not a viable molecule means that bond order is zero or negative

H_2^{2-} total no. of electrons = 4

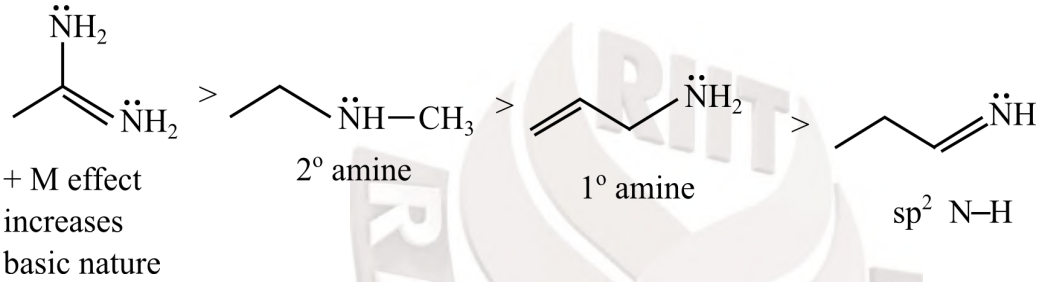
$$\therefore BO = \left[\frac{N_b - N_{Ab}}{2}\right] = \frac{2-2}{2} = 0$$

13.

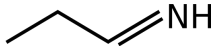
Difficulty : Easy

Topics :

GOC,



sp^2N is less basic than sp^3N

$\therefore$ 

is least basic.

14.

Difficulty : Easy

Topics :

Solid State,

In frenkel defect Cations migrate from its lattice position to interstitial sites.

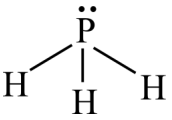
15.

Difficulty : Easy

Topics :

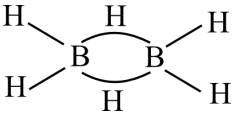
Chemical Bonding,

Ionic bond

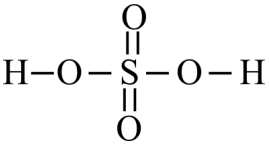


Covalent bond.

$(O = O)$ Covalent bond.



Covalent bond.



Covalent bond.

16.

Difficulty : Difficult

Topics :

Coordination Compounds,

Complex	Oxidation state of Cr	Ligand	Nature
$[\underline{C}r(H_2O)_6]Cl_3$	$+3$	H_2O	Neutral
$[\underline{C}r(C_6H_6)_2]$	0	C_6H_6	Neutral
$K_2[\underline{C}r(CN)_2(O)_2(O_2)(NH_3)]$	$+6$	O^{2-} CN^- O_2^{2-}	Anionic
		NH_3	Neutral

17.

Difficulty : Medium

Topics :

Hydrogen & Its compound,

$2H_2O_2 \longrightarrow 2H_2O + O_2$

Hence H_2O_2 can be used as oxidant as well as reductant

Acidic Medium : $[Fe(CN)_6]^{4-} + H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

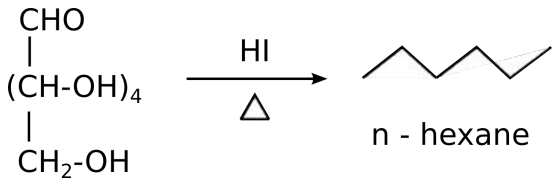
Basic Medium : $[Fe(CN)_6]^{3-} + H_2O + \bar{O}H \longrightarrow [Fe(CN)_6]^{4-} + H_2O + O_2$

18.

Difficulty : Easy

Topics :

BIOMOLECULES ,



HI is reducing agent it can reduce aldehyde and alcohols.

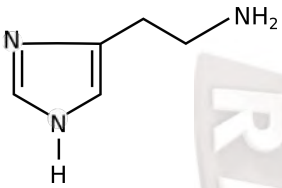
19.

Difficulty : Medium

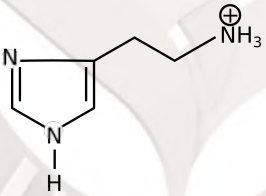
Topics :

AMINES,

Human blood is buffer solution.

Histamine is  $(pKa = 6)$

Human blood $pH > 7$,

$\therefore$ Histamine 1° amine is in protinated form 

20.

Difficulty : Easy

Topics :

Environmental Chemistry,

The F^- (fluoride) ions make the enamel on teeth much harder by converting hydroxypatite

$[3Ca_3(PO_4)_2.Ca(OH)_2]$ the enamel on the surface of the teeth, into much harder fluropatite

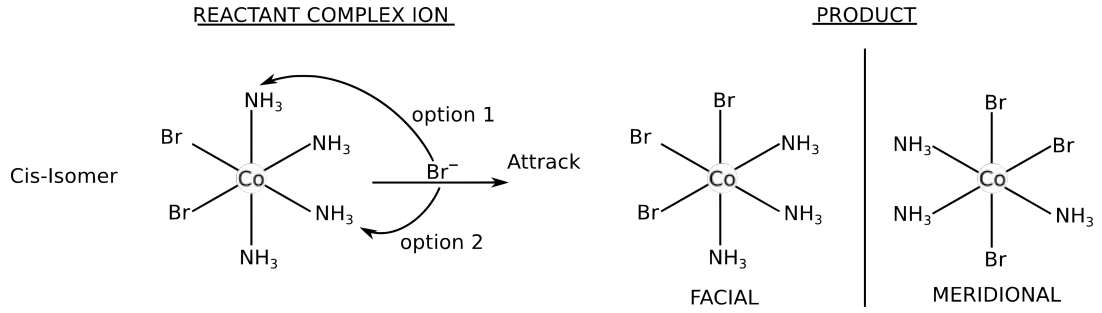
$[3Ca_3(PO_4)_2.CaF_2]$

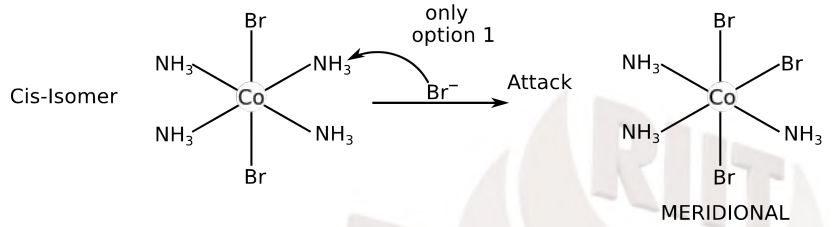
21.

Difficulty : Difficult

Topics :

Coordination Compounds,



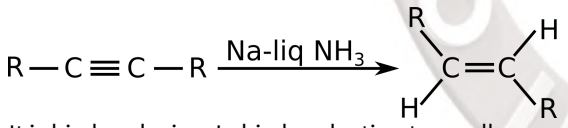


22.

Difficulty : Medium

Topics :

Hydrocarbon,



It is birch reducing. In birch reduction trans alkenes are formed.

23.

Difficulty : Difficult

Topics :

Mole concept,

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\frac{12x}{y} = \frac{6}{1} \quad \frac{x}{y} = \frac{1}{2} \quad .....(1)$$

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$$x + \frac{y}{4} - \frac{z}{2} = \frac{1}{2}\left[x + \frac{y}{4}\right]$$

$$\frac{x}{z} = \frac{2}{3}$$

$\therefore$ E.F. of compound $= x : y : z \equiv 2 : 4 : 3$

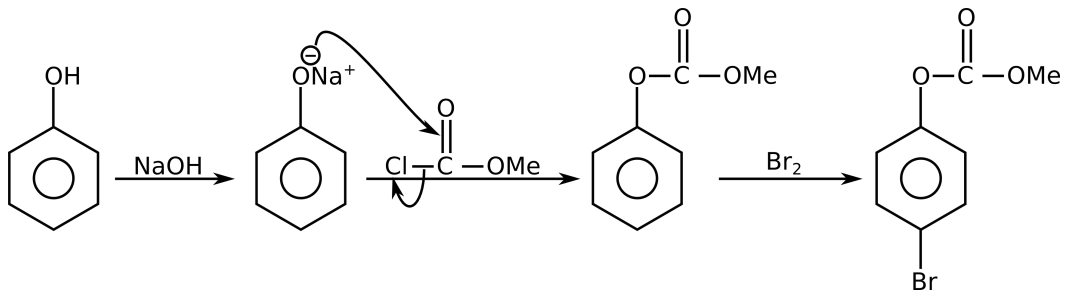
$= C_2H_4O_3$

24.

Difficulty : Medium

Topics :

Phenols,

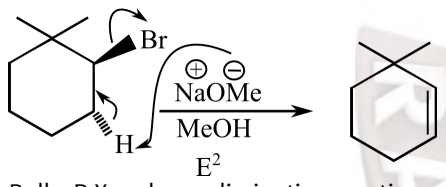


25.

Difficulty : Difficult

Topics :

Ethers,



Bulky R-X undergo elimination reaction with $R-O^{\ominus}Na^{+}$

26.

Difficulty : Medium

Topics :

Chemical Kinetics,

$k = Ae^{-\Delta H/RT}$

$ln\ k = ln\ A - \dfrac{\Delta H}{RT}$ $\quad \Delta H < 0$ (Exothermic $R \times N$)

$y = C + mx \ \Rightarrow$ Slope $= \dfrac{-\Delta H}{R} > 0$

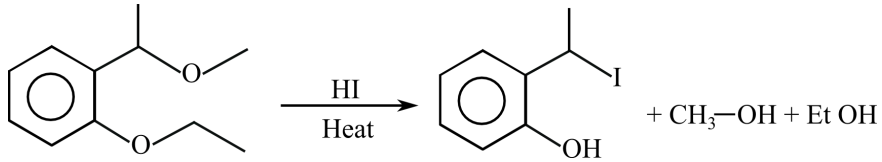
$\therefore$ graph A & B are possible

27.

Difficulty : Medium

Topics :

Ethers,



Phenolic group does not react with HI

28.

Difficulty : Difficult

Topics :
Ionic equilibrium,

When the solution begins to PPl

$[Ba^{+2}][SO_4^{2-}] = 10^{-10}$

$[Ba^{+2}]\left[\frac{50 \times 1}{500}\right] = 10^{-10}$

$\therefore [Ba^{+2}] = 10^{-9} M$ in $500\ mL$

Initial $[Ba^{+2}]$ in $450\ mL$ is

$= \frac{10^{-9} \times 500}{450}$

$= 1.1 \times 10^{-9} M$

29.

Difficulty : Medium

Topics :

Chemical Kinetics,

$-r = kp^n$

$1 = k\left(\frac{363 \times 95}{100}\right)^n$

$0.5 \times k\left(\frac{363 \times 67}{100}\right)^n$

$2 = \left(\frac{95}{67}\right)^n \Rightarrow 2 = (1.415)^n$

$2 = (\sqrt{2})^n$

$\Rightarrow n = 2$

30.

Difficulty : Medium

Topics :

Solutions,

$\Delta T_f = ik_f m$

$T_f^o - T_f = ik_p m$

$\therefore i \propto \frac{1}{F.P}$

Thus lesser the no. of particles/ions higher will be the freezing point.

31.

Difficulty : Medium

Topics :

Indefinite integration,

Divide N_r & D_r by $\cos^{10} x$

$$\Rightarrow\ I = \int \frac{\tan^2 x \sec^4 x}{(\tan^5 x + \tan^2 x + \tan^3 x + 1)^2} \sec^2 x\ dx$$

Put $\tan x = t$

$$\Rightarrow\ I = \int \frac{t^2(1+t^2)^2}{(t^5+t^2+t^3+1)^2}\ dt = \int \frac{t^2(1+t^2)^2}{(t^2+1)^2(t^3+1)^2} dt$$

$$= \int \frac{t^2}{(1+t^3)^2} dt\ = -\frac{1}{3(1+t^3)} + C$$

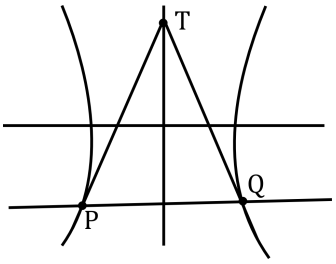
$$= \frac{-1}{3(1+\tan^3 x)} + C$$

32.

Difficulty : Medium

Topics :

Conic Sections,



PQ will be chord of contact

$\Rightarrow$ Equation of PQ is

$4x(0) - y(3) = 36$

$i.e.\ y = -12$

Hence take

$P = (\alpha, -12)$ & $Q = (-\alpha, -12)$

$\Rightarrow\ 4\alpha^2 - 12^2 = 36\ \Rightarrow\ \alpha = \pm 3\sqrt{5}$

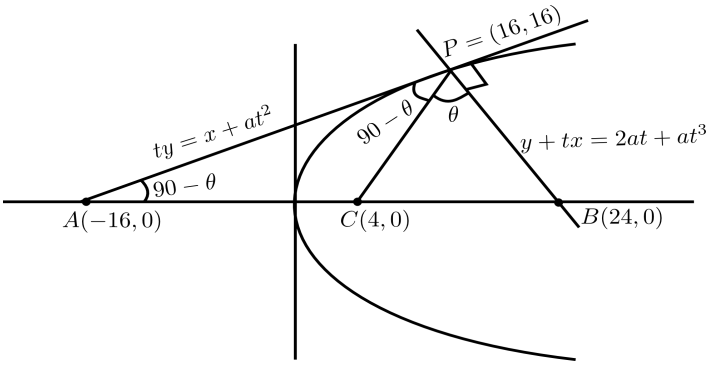
$\Rightarrow\ \ PQ = 6\sqrt{5}\ \Rightarrow \text{Area} = \frac{1}{2} \cdot 6\sqrt{5}(12+3) = 45\sqrt{5}$

33.

Difficulty : Medium

Topics :

Conic Sections,



$A = (-at^2, 0) \quad B = (2a + at^2, 0), \ t = 2$

$\Rightarrow C = (a, 0) \equiv (4, 0) \quad \text{(focus)}$

$\Rightarrow \quad \angle CPB = \theta = 90^o - \angle CAP$

$\Rightarrow \ \tan\theta = \cot(\angle CAP) = \dfrac{1}{\text{slope of tangent}} = t = 2$

Alternately $\quad : m_{CP} = \dfrac{16-0}{16-4} = \dfrac{4}{3}$

$m_{BP} = \dfrac{16-0}{16-24} = -2$

$\Rightarrow \quad \tan\theta = \left|\dfrac{m_{CP} - m_{BP}}{1 + m_{CP}m_{BP}}\right| = \left|\dfrac{\frac{4}{3} + 2}{1 - \frac{8}{3}}\right| = 2$

34.

Difficulty : Medium

Topics :

Vectors,

$\vec{a} \cdot \vec{a} = 14, \ \vec{b} \cdot \vec{b} = 2, \ \vec{a} \cdot \vec{b} = 2$

Take $\ \vec{u} = x\vec{a} + y\vec{b}$

$\vec{u} \cdot \vec{a} = 0 \qquad \Rightarrow 14x + 2y = 0 \quad \Rightarrow \quad y = -7x$

$\vec{u} \cdot \vec{b} = 24 \qquad \Rightarrow 2x + 2y = 24 \quad \Rightarrow \quad x + y = 12$

$\Rightarrow -6x = 12$

$\Rightarrow x = -2$

$\Rightarrow \ y = 14$

$\Rightarrow \quad \vec{u} = -2\vec{a} + 14\vec{b}$

$= 4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow \ |\vec{u}|^2 = 4^2(1^2 + 2^2 + 4^2) = 16 \times 21 = 336$

Alternately, vector $\perp^r$ to $\vec{a}$ & in the plane of $\vec{a}$ & $\vec{b}$ will be $\lambda \ \vec{a} \times (\vec{a} \times \vec{b}) = \lambda[(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}]$

$= \lambda[2\vec{a} - 14\vec{b}]$

$\vec{u} \cdot \vec{b} = 24 \Rightarrow \ \lambda[2(\vec{a} \cdot \vec{b}) - 14(\vec{b} \cdot \vec{b})] = -24\lambda$

$\Rightarrow \lambda = -1$

$\Rightarrow \ \vec{u} = -2\vec{a} + 14\vec{b}$

35.

Difficulty : Easy

Topics :

Complex Numbers,

$\{\alpha, \beta\} = \{-\omega, -\omega^2\}$

$\Rightarrow \quad \alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107}$

$= -(\omega^2 + \omega) = 1$

36.

Difficulty : Easy

Topics :

Area under curve,

$\alpha = \frac{\pi}{6},\ \beta = \frac{\pi}{3},\ gof(x) = \cos x,\ x > 0$

$\Rightarrow \quad \text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x dx = \frac{\sqrt{3}-1}{2}$

37.

Difficulty : Easy

Topics :

Binomial Theorem,

$(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$

$= 2\left[{}^5C_0 x^5 \ + \ {}^5C_2 x^3 (x^3 - 1) \ + \ {}^5C_4 x (x^3 - 1)^2\right]$

$\Rightarrow \quad \text{sum of coefficient of all odd degree terms} = 2\left[1 - \ {}^5C_2 \ + \ {}^5C_4 (1+1)\right]$

$= 2$

38.

Difficulty : Medium

Topics :

Sequence & Series,

$\sum_{0}^{12} a_{4k+1} = 416 = a_1 + a_5 + ... + a_{49} = 13 \cdot a_{25}$

$\Rightarrow \quad a_{25} = 32$

$a_9 + a_{43} = 66 = 2.a_{26} \Rightarrow \ a_{26} = 33$

$\Rightarrow \ a_1, a_2, a_3, ... = 8, 9, 10, ...$

$\Rightarrow \quad a_1^2 + a_2^2 + ... + a_{17}^2 = 8^2 + 9^2 + ... + 24^2 = 140m$

$\Rightarrow 140m = (1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 + ... + 7^2)$

$= \frac{24 \cdot 25 \cdot 49}{6} - \frac{7 \cdot 8 \cdot 15}{6} = \frac{7 \cdot 8 \cdot 5}{6}[3 \cdot 5 \cdot 7 - 3]$

$= \frac{7 \cdot 8 \cdot 5 \cdot 3 \cdot 34}{6} = 7 \cdot 4 \cdot 5 \cdot 34 \ \Rightarrow \ m = \frac{7 \cdot 4 \cdot 5 \cdot 34}{140} = 34$

39.

Difficulty : Easy

Topics :

STATISTICS,

Take $x_i - 5 = y_i \Rightarrow$ standard deviation of x_i will be same as standard deviation of y_i

Now $\sum y_i = 9 \Rightarrow \bar{y} = 1 = \dfrac{\sum y_i}{9}$

$\sum y_i^2 = 45 \Rightarrow \sigma^2 = \dfrac{\sum y_i^2}{9} - \left(\dfrac{\sum y_i}{9}\right)^2$

$= \dfrac{45}{9} - 1^2 = 4$

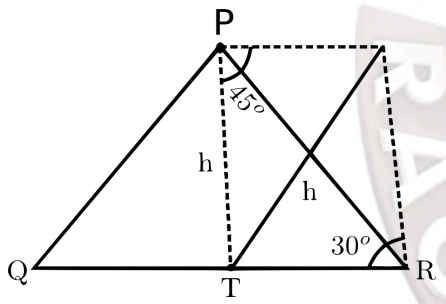
$\Rightarrow \sigma = 2$

40.

Difficulty : Easy

Topics :

Height & Distance,



If T is midpoint of $QR \Rightarrow PT = h$ & $RT = \sqrt{3}h$

Now ΔPRT is right angled

$\Rightarrow PR = 2h = 200 \Rightarrow h = 100$

41.

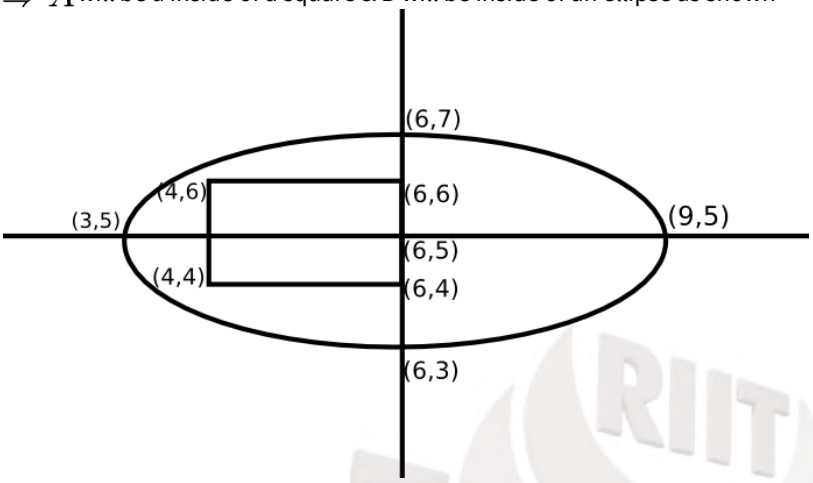
Difficulty : Medium

Topics :

Ellipse,
SETS, RELATION AND FUNCTIONS,

$a \in (4,6)\ \&\ b \in (4,6)$

$\Rightarrow A$ will be a inside of a square & B will be inside of an ellipse as shown



As point (4,4) & (4,6) are inside the ellipse

$\Rightarrow A \subset B$

42.

Difficulty : Easy

Topics :

Permutations & Combinations,

${}^6C_4.\ {}^3C_1.4! = 1080$

43.

Difficulty : Medium

Topics :

Functions,

Let $x-\dfrac{1}{x}=t\Rightarrow t\in R-\{0\}$

$$h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}-2+2}{x-\frac{1}{x}}$$

$$h(x)=\frac{\left(x-\frac{1}{x}\right)^2+2}{\left(x-\frac{1}{x}\right)}$$

$$=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

using A.M., G.M.

$$\frac{\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}}{2}\geqslant\sqrt{\left(x-\frac{1}{x}\right)\cdot\frac{2}{\left(x-\frac{1}{x}\right)}}$$

$$\Rightarrow\boxed{h(x)\geqslant 2\sqrt{2}}$$

44.

Difficulty : Medium

Topics :

Limits,

$a-1<[a]\leq a$ so,

$$(1+2+3+...+15)-15x<x.\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)\leq 1+2+3+....+15$$

$\text{where}x\in R^+$

Now taking limits on each

$$\Rightarrow\frac{15(16)}{2}-0\leq\lim_{x\to 0^+}x.\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right)\leqslant\frac{15(16)}{2}$$

$$\Rightarrow 120\leq\lim_{x\to 0^+}x.\left(\sum_{r=1}^{10}\left[\frac{r}{x}\right]\right)\leqslant 120$$

So $\displaystyle\lim_{x\to 0^+}x.\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right)=120$

45.

Difficulty : Easy

Topics :

Definite integration,

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x}{1+2^x}dx \qquad \text{____(1),}$$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x\ dx}{1+2^{-x}}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x\times\ 2^x}{1+2^x}dx \quad \text{____ (2)}$$

(using $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

(1) + (2) $\Rightarrow 2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x.\ \ (1+2^x)}{1+2^x}dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\ \ dx=2\int_0^{\frac{\pi}{2}}\sin^2 x\ dx$

$$I=\int_0^{\frac{\pi}{2}}\sin^2 x dx \quad \text{_____ (3)}$$

$$I=\int_0^{\frac{\pi}{2}}\cos^2 x\ dx \quad \text{_____ (4)}$$

$$\left(\text{using }\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

(3) + (4) $\Rightarrow I=\frac{1}{2}.\int_0^{\frac{\pi}{2}}(\sin^2 x+\cos^2 x)dx=\frac{\pi}{4}$

46.

Difficulty : Easy

Topics :

Probability,

Probability of drawn ball is red

$$=P(R\,R)+P(B\,R)$$

$$=\frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}$$

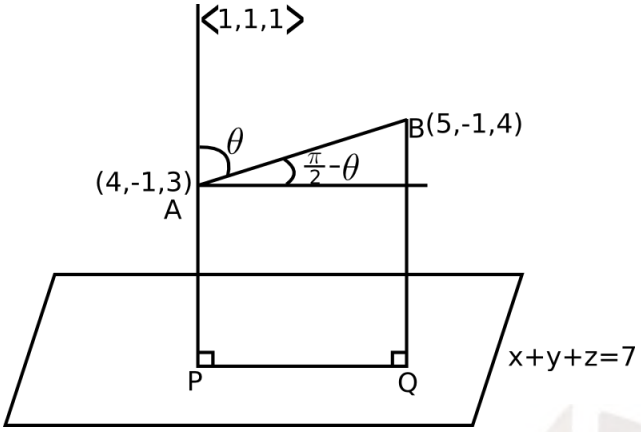
$$=\frac{48}{120}=\frac{2}{5}$$

47.

Difficulty : Easy

Topics :

Three Dimensional Geometry,
Plane,



DR of AB $< 1, 0, 1 >$
DR of normal of the plane $< 1, 1, 1 >$. If θ be the angle be made by the line segment AB with normal, then

$$\cos\theta = \frac{1+1}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

Length of projection of AB on given plane is $PQ = AB\sin\theta = \sqrt{2} \times \frac{1}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

48.

Difficulty : Medium

Topics :

Trigonometric Ratios & Identities,

$$8\cos x.\left[\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right]=1$$

$$\Rightarrow 8\cos x.\left[\left(\cos^2 x-\sin^2\frac{\pi}{6}\right)-\frac{1}{2}\right]=1$$

$$\Rightarrow 8\cos x\left[\cos^2 x-\frac{3}{4}\right]=1$$

$$\Rightarrow 2[4\cos^3 x-3\cos x]=1$$

$$\Rightarrow \cos 3x=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\Rightarrow 3x=2n\pi\pm\frac{\pi}{3},\quad n\in I$$

$$\Rightarrow x=\frac{2n\pi}{3}\pm\frac{\pi}{9},\quad n\in I$$

as $x \in [0, \pi]$, then

$$x=\frac{\pi}{9},\frac{2\pi}{3}+\frac{\pi}{9},\ \frac{2\pi}{3}-\frac{\pi}{9}$$

Hence sum of all values of $x = \frac{\pi}{9}+\frac{4\pi}{3}=\frac{13\pi}{9}$

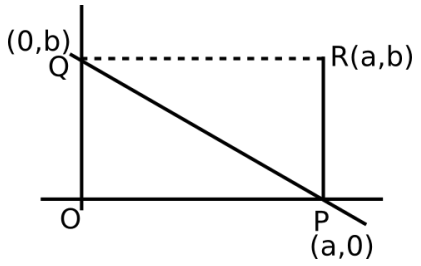
so value of k is $\frac{13}{9}$

49.

Difficulty : Easy

Topics :

Straight lines,
Locus and its Equation,



Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

as $(2,3)$ lies on the line, so $\frac{2}{a}+\frac{3}{b}=1$

hence locus of the point R is $\frac{2}{x}+\frac{3}{y}=1 \Rightarrow 3x+2y=xy$

Note that the given option is not the equation of the locus but rather a superset of it as origin does not lie on the locus but it satisfies the given option.

50.

Difficulty : Difficult

Topics :

Sequence & Series,

$A=1^2+2\times 2^2+3^2+2\times 4^2+.....+19^2+2\times 20^2$

$=(1^2+2^2+3^2+.....+20^2)+(2^2+4^2+6^2+.....+20^2)$

$=(1^2+2^2+3^2+.....+20^2)+4(1^2+2^2+3^2+.....+10^2)$

similarly $B=(1^2+2^2+3^2+.....+40^2)+4(1^2+2^2+3^2+.....+20^2)$

Now $B-2A=(1^2+2^2+3^2+.....+40^2)+2(1^2+2^2+.....+20^2)$

$-8(1^2+2^2+3^2+.....+10^2)$

$=\frac{40\times 41\times 81}{6}+2\times\frac{20\times 21\times 41}{6}-8\times\frac{10\times 11\times 21}{6}$

$=20\times 41\times 27+20\times 7\times 41-20\times 22\times 7$

$=20[41\times 34-22\times 7]$

$=20\times 1240=248\times 100$

$=100\lambda$

So $\lambda=248$

51.

Difficulty : Medium

Topics :

Application of derivatives,
Orthogonal trajectory,
Parabola,
Ellipse,

$y^2 = 6x \Rightarrow 2y\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y} = m_1$ (say)

$9x^2 + by^2 = 16 \Rightarrow 18x + 2by \ \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{9x}{by} = m_2$ (say)

as $m_1 \times m_2 = -1 \Rightarrow \frac{-27x}{by^2} = -1$

$\Rightarrow \frac{27x}{b \times 6x} = 1 \Rightarrow b = \frac{9}{2}$ (using $y^2 = 6x$)

52.

Difficulty : Easy

Topics :

Equation of Straight lines,
CO-ORDINATE GEOMETRY,

$AB = 2BC$
$\Rightarrow AC = \frac{3}{2}AB = \frac{3}{2}\sqrt{36+4} = 3\sqrt{10}$

which is the diameter of the circle so radius of the circle is $\frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}$

53.

Difficulty : Medium

Topics :

ContinuityDifferentiability & Derivatives,

$f(x) = |x-\pi|\,(e^{|x|}-1)sin\,|x|$ has a double root at $x = 0$ & at $x = \pi$

Because at $x = 0$ both $(e^{|x|}-1)$ & $sin\,|x|$ have root hence $f(x)$ will have a double root.

Similarly at $x = \pi$ & both $|x-\pi|$ & $sinx$ have root hence $f(x)$ will have a double root.

Double root implies function is differentiable at both $x = 0, \pi$. Hence S will be an empty set.

54.

Difficulty : Easy

Topics :

Determinants & Matrices,

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = \begin{vmatrix} x-4 & 2x & 2x \\ x+4 & -(x+4) & 0 \\ 0 & x+4 & -(x+4) \end{vmatrix}$$

$(R_3 \to R_3 - R_2 \;\; or \;\; R_2 \to R_2 - R_1)$

$$= (x+4)^2 \begin{vmatrix} x-4 & 2x & 2x \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$= (x+4)^2[x-4+2x+2x]$

$= (-4+5x)[x-(-4)]^2 = (A+Bx)(x-A)^2$

So $A = -4$ and $B = 5$

Hence order pair of $(A, B) \equiv (-4, 5)$

55.

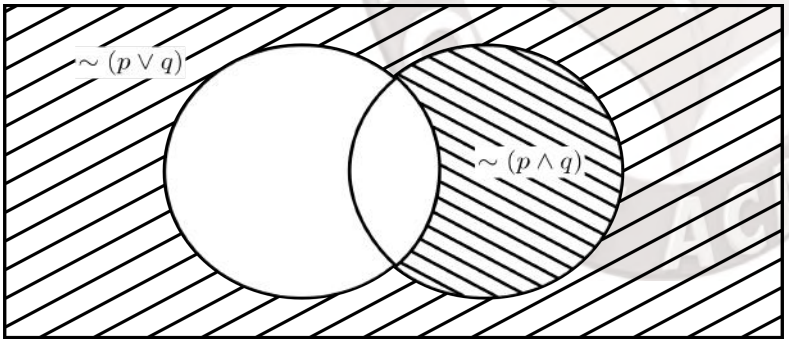
Difficulty : Easy

Topics :

Mathematical reasoning,

$\sim (p \vee q) \vee (\sim p \wedge q)$

$= \sim p$



Alternately

p	q	$\sim p$	$p \vee q$	$a = \sim (p \vee q)$	$b = \sim p \wedge q$	$a \vee b$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

From the truth table, it is clear that $\sim (p \vee q) \vee (\sim p \wedge q) = \sim p$

56.

Difficulty : Medium

Topics :

Solutions of simultaneous Equations,
Determinants & Matrices,

$x + ky + 3z = 0....(i)$
$3x + ky - 2z = 0....(ii)$
from (i) and (ii) $x + 3z = 3x - 2z \Rightarrow 2x = 5z....(iii)$
also $2x + 4y - 3z = 0$
$\Rightarrow 2z + 4y = 0 \Rightarrow y = -\frac{z}{2}.....(iv)$ using (iii)
Now, $\frac{xz}{y^2} = \frac{\frac{5}{2}z^2}{\frac{z^2}{4}} = 10$

57.

Difficulty : Medium

Topics :

Quadratic Equations,
Modulus Function,

$2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$

case I : $\sqrt{x} \geq 3 \;\Rightarrow\; x \geq 9$

$\Rightarrow 2(\sqrt{x} - 3) + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$

$\Rightarrow 2\sqrt{x} - 6 + (\sqrt{x})^2 - 6\sqrt{x} + 6 = 0$

$\Rightarrow (\sqrt{x})^2 - 4\sqrt{x} = 0$

$\sqrt{x} = 0, 4$

$\Rightarrow x = 0, 16 \text{ (0 rejected)}$

Case II : $\sqrt{x} < 3$

Now, $-2\sqrt{x} + 6 + (\sqrt{x})^2 - 6\sqrt{x} + 6 = 0$

$(\sqrt{x})^2 - 8\sqrt{x} + 12 = 0$

$(\sqrt{x} - 2)(\sqrt{x} - 6) = 0$

$x = 4, 36 \text{ (36 rejected)}$

so finally $x = 4, 16$, two solutions.

58.

Difficulty : Medium

Topics :

Application of derivatives,
Circles,
Condition of tangency,
Parabola,

Slope of tangent to $x^2 = y - 6$ at $(1,7)$ will be $\left.\frac{dy}{dx}\right\}_{(1,7)} = 2x\}_{(1,7)} = 2$

$\Rightarrow$ equation of tangent will be $y = 2x + 5$

which touches the circle $(x+8)^2 + (y+6)^2 = 100 - c$

$\Rightarrow$ distance of $y = 2x + 5$ from $(-8,-6)$ will be radius $= \sqrt{100-c}$

$$\Rightarrow \left|\frac{2(-8)-(-6)+5}{\sqrt{2^2+1^2}}\right| = \sqrt{100-c}$$

$$\Rightarrow c = 95$$

59.

Difficulty : Easy

Topics :

Differential Equations,

$$\frac{d}{dx}(y\sin x) = 4x$$

$\Rightarrow y\sin x = 2x^2 + c$ As $y\left(\frac{\pi}{2}\right) = 0 \Rightarrow c = -\frac{\pi^2}{2}$

$$\Rightarrow y\sin x = 2x^2 - \frac{\pi^2}{2}$$

$$\Rightarrow y\left(\frac{\pi}{6}\right) = -\frac{8}{9}\pi^2$$

60.

Difficulty : Difficult

Topics :

Three Dimensional Geometry,
Family of Planes,

The plane will pass through the intersection of $2x-2y+3z-2=0$ & $x-y+z+1=0$.
Hence its equation will be $(2x-2y+3z-2)+\lambda(x-y+z+1)=0$
i.e. $(2+\lambda)x-(2+\lambda)y+(3+\lambda)z-2+\lambda=0....(i)$
Similarly this plane will pass through the intersection of $x+2y-z-3=0$ & $3x-y+2z-1=0$
hence its equation will be
$(x+2y-z-3)+\mu(3x-y+2z-1)=0$
i.e. $(1+3\mu)x+(2-\mu)y+(2\mu-1)z-(3+\mu)=0.....(ii)$
as (i) & (ii) represent same plane

$$\Rightarrow \frac{2+\lambda}{1+3\mu}=\frac{2+\lambda}{\mu-2}=\frac{3+\lambda}{2\mu-1}=\frac{\lambda-2}{-(3+\mu)}$$

solving $\Rightarrow \mu=-\frac{3}{2}$

$\Rightarrow$ equation of plane is $7x-7y+8z+3=0$

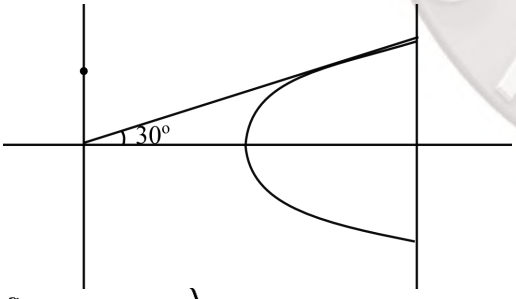
whose distance from origin $=\left|\frac{3}{\sqrt{7^2+7^2+8^2}}\right|=\frac{1}{3\sqrt{2}}$

61.

Difficulty : Medium

Topics :

Wave optics,



$$\frac{a}{2}\sin 30^\circ=\frac{\lambda}{2}$$

$$\frac{a}{2}=\lambda \qquad a=2\lambda \qquad \lambda=\frac{a}{2}=\frac{10^{-6}}{2}$$

$$\beta=\frac{\lambda D}{d}$$

$$\frac{1}{100}=\frac{10^{-6}}{2d}\times\frac{1}{2} \qquad \Rightarrow d=25\mu m$$

62.

Difficulty : Difficult

Topics :

Modern Physics,

$$DE = E_n - E_g$$
$$\frac{hc}{\Lambda} = E_g - E_n$$
$$\frac{hc}{\Lambda} = \frac{P_g^2}{2m} - \frac{P_n^2}{2m}$$
$$\frac{hc}{\Lambda} = \frac{P_g^2}{2m}\left[1 - \left(\frac{P_n}{P_g}\right)^2\right]$$
$$\frac{\Lambda}{hc} = \frac{2m}{P_g^2}\left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$
$$\frac{\Lambda}{hc} \approx \frac{2m}{P_g^2}\left[1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]$$
$$\Lambda = A + \frac{B}{\lambda_n^2}$$

63.

Difficulty : Easy

Topics :

Semiconductors,

$$I = \frac{\Delta V}{R} = \frac{3 - 0.7}{200}$$
$$I = 0.0115$$
$$I = 11.5\,mA$$

64.

Difficulty : Easy

Topics :

Errors,

$$\rho = M\,L^{-3}$$
$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$$
$$\%\rho = 1.5 + 3$$
$$\%\rho = 4.5\,\%$$

65.

Difficulty : Medium

Topics :

MAGNETISM,

$$r = \frac{\sqrt{2mKE}}{qB}$$

$$r_e : r_p : r_\alpha$$

$$\frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} = \frac{\sqrt{4m_p}}{2e}$$

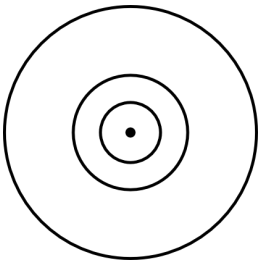
$$\therefore\ r_p : r_\alpha > r_e$$

66.

Difficulty : Medium

Topics :

Electrostatics,



$$V_B = \frac{K(Q_A + Q_B)}{b} + \frac{KQ_C}{c}$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + \frac{c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

67.

Difficulty : Medium

Topics :

Friction,

$$m.g = \text{Friction}_{\max}$$

$$m.g = \mu(m_2 + m)g$$

$$\frac{100}{3} = 10 + m$$

$$m = 23.33 \quad \therefore\ m_{\min} = 27.3\,Kg$$

68.

Difficulty : Medium

Topics :

Work ,Energy and power,

$$U = -\frac{K}{2r^2} \qquad \frac{dU}{dr} = \frac{K}{r^3}$$

$$F = -\frac{K}{r^3}$$

$$F = \frac{mv^2}{r}$$

$$\frac{K}{r^3} = \frac{mv^2}{r} \qquad \frac{mv^2}{2} = \frac{K}{2r^2}$$

$$TE = PE + KE = -\frac{K}{2r^2} + \frac{K}{2r^2} = 0$$

69.

Difficulty : Medium

Topics :

Capacitors,

$$q^1 = KCV_0 - CV_0$$
$$= (K-1)CV_0$$
$$= \left(\frac{5}{3} - 1\right) 90 \times 10^{-12} \times 20$$
$$= 1.2 \times 10^{-9} C$$
$$= 1.2\, n\, C$$

70.

Difficulty : Easy

Topics :

Simple harmonic motion,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad f^2 = \frac{1}{4\pi^2}\frac{K}{m}$$

$$K = 4\pi^2 f^2 m$$

$$K = 4 \times (3.14)^2 \times (10^{12})^2 m$$

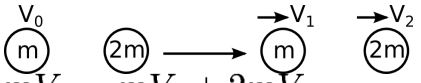
$$K = 7.1\, N/m$$

71.

Difficulty : Medium

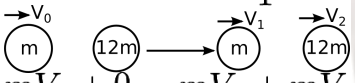
Topics :

Collisions,



$$mV_0 = mV_1 + 2mV_2$$
$$V_0 = V_2 - V_1$$
$$V_0 = V_1 + 2V_2$$
$$V_0 = V_2 - V_1$$
$$2V_0 = 3V_2$$
$$\frac{2V_0}{3} = V_2$$
$$V_1 = V_2 - V_0$$
$$V_1 = \frac{2V_0}{3} - V_0$$
$$V_1 = \frac{-V_0}{3}$$

% fractional loss = $\frac{\left(\frac{1}{9} - 1\right)}{1} = -\frac{8}{9} \approx -0.888$



$$mV_0 + 0 = mV_1 + mV_2$$
$$V_0 = V_2 - V_1$$
$$V_0 = V_1 + 12V_2$$
$$V_0 = V_2 - V_1$$
$$2V_0 = 13V_2$$
$$2V_0 = V_2$$
$$V_1 = V_2 - V_0$$
$$= 2V_0 - V_0$$
$$V_1 = \frac{11}{13}V_0$$

$\therefore$ % loss $\frac{49}{81} - 1$

$$= \frac{49 - 81}{81}$$

$$= \frac{121}{169} - 1 = -\left(\frac{169 - 121}{169}\right) = -0.28$$

72.

Difficulty : Easy

Topics :

MAGNETISM,

$$m = I\pi R^2 \quad B_1 = \frac{\mu_0 I}{2R}$$

$$2m = I\pi\ R_f^2$$

$$\boxed{R_f = \sqrt{2}R}$$

$$B_2 = \frac{\mu_0 I}{2\sqrt{2}R}$$

$$\frac{B_1}{B_2} = \sqrt{2}$$

73.

Difficulty : Medium

Topics :

Current electricity,

$$\frac{5 + r_0}{5} = \frac{52}{40}$$

$$1 + \frac{r_0}{5} = 1 + \frac{12}{40}$$

$$r_0 = \frac{60}{40} = \frac{3}{2}$$

74.

Difficulty : Easy

Topics :

Communication systems,

$$\text{10\% of} 10\ GHz = \frac{10}{100} \times 10 \times 10^9$$

$$= 10^9\ Hz$$

$$n \times 5 \times 10^3 = 10^9$$

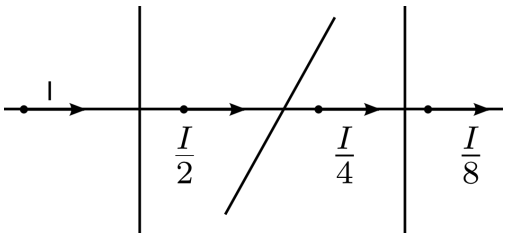
$$n = \frac{10^6}{n} = 2 \times 10^5$$

75.

Difficulty : Medium

Topics :

Wave optics,



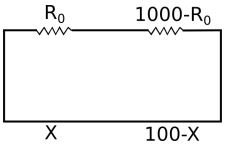
$$\frac{I}{8} = \frac{I}{2}\cos^2\phi\sin^2\phi$$

$$\therefore \phi = 45^\circ$$

76.

Difficulty : Medium

Topics :
Current electricity,



$$\frac{R_0}{1000 - R_0} = \frac{X}{100 - X}$$

$$X - (100 - X) = 10 \qquad 2X = 110$$

$$X = 55$$

$$\therefore R_0 = 550\Omega$$

77.

Difficulty : Medium

Topics :

Rotational Motion,

$$I_R = I_{Total} - I_{removed}$$

$$= \frac{9MR^2}{2} - \left[\frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{M4R^2}{9}\right]$$

$$I_R = 4MR^2$$

78.

Difficulty : Medium

Topics :

Collisions,

kinetic energy is 50%

$$k_f = \frac{3}{2}k_i$$

$$\frac{1}{2}2m\frac{V_0^2}{4} + \frac{1}{2}\frac{mV_{rel}^2}{2} = \frac{3}{2}\left[\frac{1}{2}2m\frac{V_0^2}{4} + \frac{1}{2}\frac{m}{2}V_0^2\right]$$

$$V_{rel}^2 = 2V_0^2$$

$$V_{rel}^0 = \sqrt{2}V_0$$

79.

Difficulty : Medium

Topics :
Electromagnetic waves,

$$C = \frac{1}{\sqrt{\mu_0\epsilon_{r_1}\epsilon_0}} \quad 2 = \sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

$$V = \frac{1}{\sqrt{\mu_0\epsilon_{r_2}\epsilon_0}}$$

$$4 = \frac{\epsilon_{r_2}}{\epsilon_{r_1}} \qquad \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

80.

Difficulty : Easy

Topics :

Alternating current,

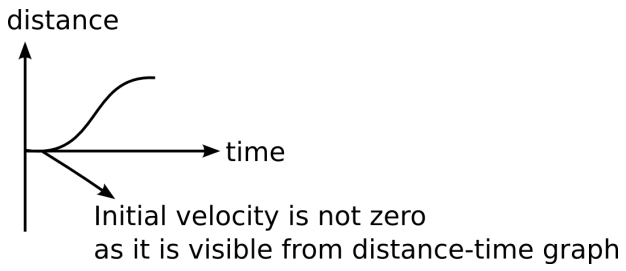
Quality factor $= \frac{X_L}{R} = \frac{W_0 L}{R}$

81.

Difficulty : Medium

Topics :

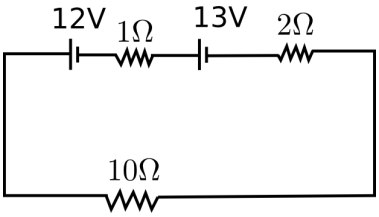
Kinematics,



82.

Difficulty : Easy

Topics :
Current electricity,



$$E_{eq} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}$$

$$V = \frac{\frac{37}{3}}{10 + \frac{2}{3}} \times 10$$

$$V = 11.56 \text{ volt}$$

83.

Difficulty : Medium

Topics :
Gravitation,

$$F = ma_r$$
$$\frac{c}{R^n} = m\left(\frac{2\pi}{T}\right)^2 R$$
$$T^2 = c_1 R^{n+1}$$
$$T \propto R^{\frac{n+1}{2}}$$

84.

Difficulty : Medium

Topics :

Modern Physics,

$$h\nu_L = 13.6$$

$$h\nu_L^{'} = \frac{13.6}{5^2}$$

$$\frac{\nu_L^{'}}{\nu_L} = \frac{1}{25}$$

$$\nu_L^{'} = \frac{\nu_L}{25}$$

85.

Difficulty : Medium

Topics :

Alternating current,

$$z = \frac{100}{20} = 5 \qquad R = X_L$$

$$5 = \sqrt{R^2 + X_L^2} \qquad R = \frac{5}{\sqrt{2}}$$

$$5 = \sqrt{R^2 + R^2} = \sqrt{2}R$$

$$\text{Average power} = i_{rms}^2 R$$

$$= \left(\frac{20}{\sqrt{2}}\right)^2 \frac{5}{\sqrt{2}}$$

$$= (10\sqrt{2})^2 \times \frac{5}{\sqrt{2}}$$

$$= \frac{200 \times 5}{\sqrt{2}} \qquad = \frac{1000}{\sqrt{2}}$$

$$\text{Wattless current} = I \sin\phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10$$

86.

Difficulty : Medium

Topics :

Thermodynamics,

(a)

$$T_2 V_2^{r-1} = T_1 V_1^{r-1}$$

$$T_2 (2V_0)^{2/3} = 300 V_0^{2/3}$$

$$T_2\ 2^{2/3} = 300$$

$$T_2 = \frac{300}{2^{2/3}} = \frac{300}{2^{0.6667}} = \frac{300}{1.5887767}$$

$$T_2 = 188.82451K$$

(b)

$$\Delta V = n\frac{3}{2}R(T_f - T_i)$$

$$= 2 \times \frac{3}{2} \times \frac{25}{3}[189 - 300]$$

$$= -25[111]$$

$$= -2.775\ kJ$$

87.

Difficulty : Easy

Topics :

Elasticity,

$$K = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$K = \frac{-mg}{-a\frac{3\Delta r}{r}}$$

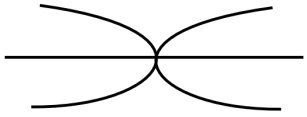
$$\frac{\Delta r}{r} = \frac{mg}{3aK}$$

88.

Difficulty : Medium

Topics :

Oscillations and Waves,



$$\frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = 2L$$

$$V = \sqrt{\frac{Y}{\rho}}$$

$$f = \frac{V}{\lambda}$$

$$f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$$

$$f \approx 5000Hz$$

89.

Difficulty : Medium

Topics :

Kinetic theory of gases,

$$F = \frac{n.2mV\cos 45^\circ}{\text{area}}$$

$$F = \frac{10^{23} \times 2 \times 3.32 \times 10^{27} \times 10^3}{2 \times 10^{-4}} \times \frac{1}{\sqrt{2}}$$

$$F = \frac{3.32}{1.414} \times 10^3 = 2.34778 \times 10^3$$

$$F = 2.35 \times 10^3 N/m^2$$

90.

Difficulty : Medium

Topics :

Rotational Motion,

$$I_{cm} = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M(2R)^2\right]$$
$$= \frac{MR^2}{2} + 3MR^2 + 24MR^2$$
$$= \frac{MR^2}{2} + 27MR^2$$
$$= \frac{55}{2}MR^2$$

$$I_P = \frac{55}{2}MR^2 + 7M \times 9R^2$$
$$= \left(\frac{55}{2} + 63\right)MR^2$$
$$= \left(\frac{55 + 126}{2}\right)MR^2$$
$$= \frac{181}{2}MR^2$$