Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

SOLUTIONS

Joint Entrance Exam | IITJEE-2018

Paper Code - B

8th April 2017 | 9.30 AM – 12.30 PM

Joint Entrance Exam | JEE Mains 2018

PART-A	PHYSICS

1.(3) For collision with deuterium:

$mv + o = mv_1 + 2mv_2$ (Conservation of momentum) .... (1)

$v_2 - v_1 = v$ $(\because e = 1)$ .... (2)

By (1) and (2) $v_1 = -\dfrac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

For collision with carbon Nucleus

$mv + 0 = mv_1 + 12mv_2$ (Conservation of momentum) .... (1)

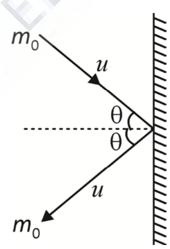
$v = v_2 - v_1$ $(\because e = 1)$ .... (2)

By (1) and (2)

$v_1 = -\dfrac{11}{13}v$

$$Pc = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

2.(3) Change in momentum of a single molecule.



$$\Delta P_0 = m_0\frac{u}{\sqrt{2}} \times 2$$

Total change in momentum per second

$$\Delta P = n\Delta P_0 = n.m_0 u\sqrt{2}$$

Pressure $= \dfrac{F}{A} = \dfrac{nm_0 u\sqrt{2}}{A}$

Substituting values: $P = 2.35 \times 10^3 N/m^2$.

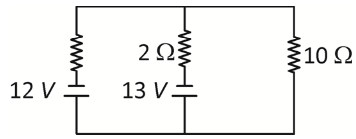
3.(1) $B = \left|\dfrac{\Delta P}{\frac{\Delta V}{V}}\right|$

$\therefore$ $\dfrac{\Delta V}{V} = \dfrac{\Delta P}{B}$

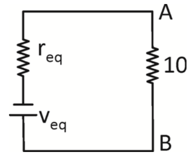
Also, $\dfrac{\Delta V}{V} = 3\dfrac{\Delta r}{r}$ (As $\Delta V = 4\pi r^2 \Delta r$)

$\Rightarrow$ $\dfrac{\Delta r}{r} = \left(\dfrac{\Delta P}{3B}\right) = \dfrac{mg}{3Ka}$

4.(4)



Equivalent circuit is



Where,

$$r_{eq} = \frac{1\times 2}{1+2} = \frac{2}{3}$$

$$v_{eq} = \frac{\left(\frac{V_1}{r_1} + \frac{V_2}{r_2}\right)}{\frac{1}{r_1} + \frac{1}{r_2}} = 12.33$$

$\therefore \quad V_{AB} = V_{eq}\dfrac{10}{10 + r_{eq}}$ = 11.55 volts

5.(1) $U = -\dfrac{K}{2r^2}$

$$F = -\frac{dU}{dr}\hat{r} = -\frac{K}{r^3}\hat{r}$$

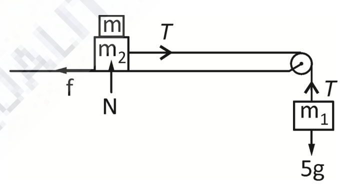
$\therefore \quad \dfrac{K}{r^3} = \dfrac{mv^2}{r}$ (As Force towards center $= \dfrac{mv^2}{r}$)

$\therefore \quad$ K. E. = $\dfrac{1}{2}mV^2 = \dfrac{K}{2r^2}$

Total energy = KE + PE $= \dfrac{K}{2r^2} - \dfrac{K}{2r^2}$ = Zero

6.(4) For m_1 to be at rest

$$T = 5g$$



For m & m_2 to be at rest

$$f = T = 5g$$

$f \le \mu(N)$

$\Rightarrow f \le 0.15(m + m_2)g$

$\Rightarrow \quad m \ge 23.33\ kg$

Amongst the options minimum mass that can be kept for no motion is 27.3 kg

7.(2) For Series limit of Lyman : $n_1 = 1$ and $n_2 = \infty$

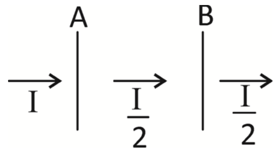
$$\Rightarrow \quad \nu_P = RcZ^2\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

For Series limit of Pfund: $n_1 = 5$ and $n_2 = \infty$

$$\Rightarrow \quad \nu_P = RcZ^2\left(\frac{1}{25} - \frac{1}{\infty}\right) = \frac{\nu_L}{25}$$

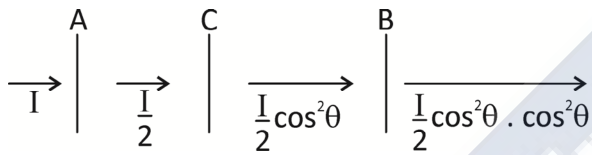
8.(1) When an unpolarized light of intensity I passes through a polarizer for the 1st time, intensity of output is $\frac{I}{2}$ (irrespective of orientation of polarizer)

So,



i.e., polarizers A and B have axes parallel to each other.

Now let the axis of C make an angle θ with A, and $(-\theta)$ with B.



$$\frac{I}{2}\cos^4\theta = \frac{I}{8}$$

Solving, $\theta = 45°$

9 (3). $\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

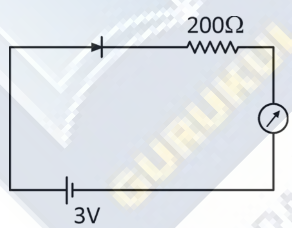
Since n is very large, using binomial

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

$$\Lambda_n = A + \frac{B}{\lambda_n^2} \qquad \text{As } \lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$$

10 (1).



Voltage across Si diode in forward bias is 0.7 volts. Hence voltage across $200\,\Omega$ resister is $3 - 0.7 = 2.3V$

$$\therefore \qquad I = \frac{2.3}{200} = 11.5 \text{ mA}$$

11.(4) $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

$$r_e = \frac{\sqrt{2m_eK}}{eB}$$

$$r_\alpha = \frac{\sqrt{2\times 4m_pK}}{2eB}$$

$$r_p = \frac{\sqrt{2 \times m_p K}}{eB}$$

Comparing (1), (2) and (3)

$$r_e < r_\alpha = r_p$$

12.(3) $q_i = CV$

$q_f = KCV$

$$q_{induced} = q_f - q_i = (K-1)CV = \left(\frac{5}{3} - 1\right) \times 90 \times 10^{-12} \times 20 = 1.2\,nC$$

13.(3) Quality factor $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$

14.(1) Overall bandwidth use for transmission $= 10\%$ of ν_C

$$\text{Number of telephonic channel } = \frac{\text{Total bandwidth}}{\text{Channel bandwidth}} = \frac{\frac{10}{100} \times 10 \times 10^9}{5 \times 10^3} = 2 \times 10^5$$

15.(3) $f = \frac{c}{\lambda} = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$

$$= \frac{1}{2 \times 0.6}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$$= \frac{1}{1.2}\sqrt{\frac{9.27 \times 10^7}{2.7}}$$

$= 4.88 \times 10^3\,Hz$

$\approx 5\,kHz$

16.(2) $$I_0 = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M(2R)^2\right] = MR^2\left[\frac{1}{2} + 3 + 24\right] = \frac{55}{2}MR^2$$

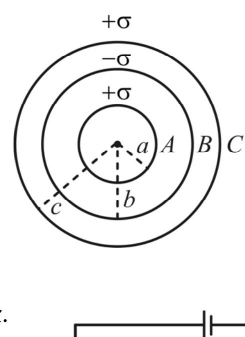
O is the centre of mass of the system. Applying parallel axis theorem between O & P.

$$I_P = I_0 + 7M(3R)^2 = \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

17.(4) $V_B = \frac{kQ_A}{b} + \frac{kQ_B}{b} + \frac{kQ_C}{c}$

$$= k\left[\frac{(+\sigma)(4\pi a^2)}{b} + \frac{(-\sigma)(4\pi b^2)}{b} + \frac{(+\sigma)(4\pi c^2)}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\sigma 4\pi\left[\frac{a^2}{b} - \frac{b^2}{b} + c\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$



18.(4) Let potential difference per unit length of potentiometer wire be x.

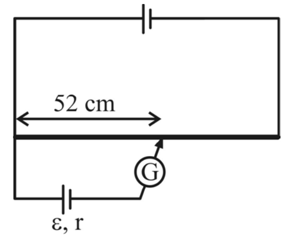
In case-I

$\varepsilon = (52)(x)$ …(i)

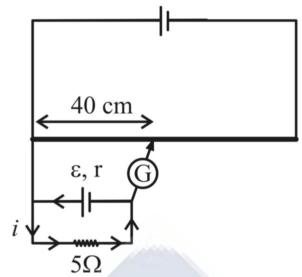
In case-II

$$i = \frac{\varepsilon}{r + 5}$$

$\varepsilon - ir = (40)(x)$



$$\varepsilon-\left(\frac{\varepsilon}{r+5}\right)r=40x \quad\Rightarrow\quad \varepsilon\left(\frac{5}{r+5}\right)=40x \qquad \ldots\text{(ii)}$$



From (i) & (ii)

$$52x=40x\left(\frac{r+5}{5}\right)$$

$$\frac{52}{40}=\frac{r}{5}+1 \quad\Rightarrow\quad r=5\left(\frac{13}{10}-1\right)=\frac{15}{10}=1.5\Omega$$

19.(1) From wave equations :

In air: $\omega=2\pi\nu,\ k=\dfrac{2\pi\nu}{c}$

In medium: $\omega=kc,\ \ k'=2k$

$$k=\frac{\omega}{c},\quad k'=\frac{\omega}{c'} \quad\Rightarrow\quad \frac{\omega}{c'}=\frac{2\omega}{c} \quad\Rightarrow\quad c'=\frac{c}{2}$$

$$\frac{1}{\sqrt{\mu_0\mu_{r_2}\in_0\in_{r_2}}}=\frac{1}{\sqrt{\mu_0\mu_{r_1}\in_0\in_{r_1}}}\frac{1}{2}$$

$\because$ Medium and air are non-magnetic

$$\mu_{r_1}=1\ ;\ \mu_{r_2}=1$$

$$\therefore\quad \frac{1}{\in_{r_2}}=\frac{1}{4\in_{r_1}} \quad\Rightarrow\quad \frac{\in_{r_1}}{\in_{r_2}}=\frac{1}{4}$$

20.(3) Angular width of central maxima $=\dfrac{2\lambda}{a}$; (where a is slit width and λ is wavelength)

$$\frac{2\lambda}{a}=\frac{\pi}{3} \qquad \ldots\text{(i)}$$

In YDSE, fringe width

$$\beta=\frac{\lambda D}{d}$$ [where d is slit separation and D is distance of screen from slits)

$$\beta=\frac{D}{d}\times\frac{\pi}{6}a \quad\Rightarrow\quad d=\frac{D\pi a}{6\beta} \quad\Rightarrow\quad d=\frac{1}{2}\times\frac{3.14\times10^{-6}}{6\times10^{-2}}\approx\frac{100}{4}\times10^{-6}\approx 25\mu m$$

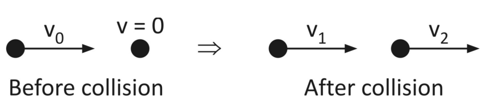
21.(4) $10^{12}=f=\dfrac{1}{T}=\dfrac{\omega}{2\pi} \quad\Rightarrow\quad \omega=2\pi\times10^{12}$

$$\because\quad \omega^2=\frac{k}{m} \quad\Rightarrow\quad k=m\omega^2=\left(\frac{108}{6.023\times10^{23}}\right)\times10^{-3}\times\left(2\pi\times10^{12}\right)^2=7.1$$

22.(3) $I=\dfrac{(9M)R^2}{2}-\left\{\dfrac{M\left(\dfrac{R}{3}\right)^2}{2}+M\left(\dfrac{2R}{3}\right)^2\right\}$

$$I=\frac{9MR^2}{2}-\left\{\frac{MR^2}{18}+\frac{4MR^2}{9}\right\}=4MR^2$$

23.(4)



It is given that final total kinetic energy has increased, so some internal energy of the system must have been converted into kinetic energy.

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 1.5\left(\frac{1}{2}mv_0^2\right)$$

$\Rightarrow \quad v_1^2 + v_2^2 = 1.5v_0^2$ ...... (i)

Since, there is no external force, momentum can be conserved

$$mv_1 + mv_2 = mv_0$$

$\Rightarrow \quad v_1 + v_2 = v_0$ ...... (ii)

From (i) & (ii)

$$v_1 + (v_0 - v_1)^2 = 1.5v_0^2$$

$$2v_1^2 - 2v_0v_1 - 0.5v_0^2 = 0$$

Relative velocity $= |v_2 - v_1| =$ Difference of roots $= \frac{\sqrt{D}}{a} = \sqrt{2}\,v_0$

24 (1). $B_1 = \frac{\mu_0 I}{2r_1}; \;\; m_1 = I\left(\pi r_1^2\right)$

$B_2 = \frac{\mu_0 I}{2r_2}; \;\; m_2 = I\left(\pi r_2^2\right)$

$\frac{m_2}{m_1} = \frac{r_2^2}{r_1^2} \quad \Rightarrow \quad 2 = \left(\frac{r_2}{r_1}\right)^2 \quad \Rightarrow \quad \frac{r_2}{r_1} = \sqrt{2}$

$\therefore \quad \frac{B_1}{B_2} = \frac{r_2}{r_1} = \sqrt{2}$

25 (1). $\rho = \frac{M}{V} = \frac{M}{L^3}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L}$

$\therefore$ Maximum % error in density $= 1.5\% + 3(1\%) = 4.5\%$

26.(1) Let the resistances in left and right slot be r and $1000 - r$ respectively

Initial: $r(100 - x) = (1000 - r)(x)$ ....(1)

After interchanging: $(1000 - r)[100 - (x - 10)] = r(x - 10)$

$(1000 - r)(110 - x) = r(x - 10)$ ....(2)

From (1): $100r - rx = 1000x - rx \quad \Rightarrow \quad r = 10x$

From (2): $(1000 - r)\left(110 - \frac{r}{10}\right) = r\left(\frac{r}{10} - 10\right)$

$\Rightarrow \quad (1000 - r)(1100 - r) = r^2 - 100r$

$\Rightarrow \quad 1000 \times 1100 - 2100r + r^2 = r^2 - 100r \quad \Rightarrow \quad r = \frac{1000 \times 1100}{2000} = 550\,\Omega$

27.(4) $\langle P \rangle = V_{rms} I_{rms} \cos\phi \; ; \qquad \phi = -\frac{\pi}{4}$

$\langle P \rangle = \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$

Wattless current, $I = I_{rms} \; \sin\phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10$

28.(4) The (1), (2) and (3) graphs can represent the motion of a ball that is thrown in vertically upward direction. Initially speed decreases, becomes zero and then on the return trip, speed increases. Slope of graph in option (4) does not explain it.

29.(1) For mono atomic gas $\gamma = \frac{5}{3}$

Using $TV^{\gamma-1}$ = constant

$$(300)V^{2/3} = (T)(2V)^{2/3} \quad \Rightarrow \quad T = \frac{300}{(2)^{2/3}} \approx 189\,K$$

$$\Delta U = n\frac{3}{2}R\Delta T = 2\left(\frac{3}{2}\times 8.314\right)(189-300) = -2768 \approx -2.7 \text{ kJ}$$

30.(1) $F \propto \frac{1}{R^n}$

$$F = \frac{k}{R^n} = \frac{mV^2}{R} \quad \Rightarrow \quad V^2 = \frac{k}{mR^{n-1}} \quad \Rightarrow \quad V \propto R^{\frac{(1-n)}{2}}$$

Now $T = \frac{2\pi R}{V} \propto \frac{R}{R^{\frac{(1-n)}{2}}} \propto R^{\frac{n+1}{2}}$

PART-B	MATHEMATICS

31.(2) $x=\frac{y+7}{2}-6 \quad \Rightarrow \quad 2x=y+7-12 \quad \Rightarrow \quad 2x=y-5 \quad \Rightarrow \quad 2x-y+5=0$

Also, centre of the circle is $(-8,\,-6)$ and the radius is $\sqrt{64+36-c}$

$\Rightarrow \quad \left(\frac{-16+6+5}{\sqrt{5}}\right)=\sqrt{100-c} \quad \Rightarrow \quad \sqrt{5}=\sqrt{100-c} \quad \Rightarrow \quad c=95$

32.(4) $(2+\lambda)x-(2+\lambda)y+(3+\lambda)z-2+\lambda=0$

$(1+3\mu)x+(2-\mu)y+(2\mu-1)z-3-\mu=0$

$\Rightarrow \quad \frac{2+\lambda}{1+3\mu}=\frac{-(2+\lambda)}{2-\mu} \quad \Rightarrow \quad \mu-2=1+3\mu \quad \Rightarrow \quad 2\mu=-3 \quad \Rightarrow \quad \mu=\frac{-3}{2}$

So the equation of plane is $7x-7y+8z+3=0$

Now, distance from origin equal to $\left|\frac{3}{\sqrt{7^2+7^2+8^2}}\right|=\frac{1}{3\sqrt{2}}$

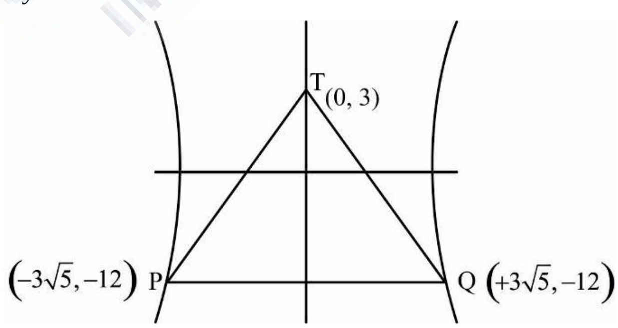
33.(1) $x^2-x+1=0 \quad \Rightarrow \quad x=\frac{1\pm\sqrt{-3}}{2}=-\omega,-\omega^2$ (where ω and ω^2 are non-real cube roots of unity)

$\Rightarrow \quad \alpha=-\omega$ and $\beta=-\omega^2$

$\Rightarrow \quad (-\omega)^{101}+\left(-\omega^2\right)^{107}=-\left(\omega^{101}+\omega^{214}\right)=-\left(\omega^2+\omega\right)=1$

34.(3) Equation of PQ,

$4x\cdot(0)-3y=36$



$y = -12$

Area of $\Delta TPQ = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$

35.(2) $2yy' = 6$

$y' = \frac{6}{2y} = \frac{3}{y_1}$

$18x_1 + 2by_1 y' = 0$

$y' = -\frac{18x_1}{2by_1} = \frac{-9x_1}{by_1} \quad \Rightarrow \quad -\frac{27x_1}{by_1^2} = -1 \quad \Rightarrow \quad b = \frac{27x_1}{y_1^2}$

$y_1^2 = 6x_1 \quad \Rightarrow \quad b = \frac{9}{2}$

36.(4) $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow \quad k = \frac{7}{2}$

$x + ky + 3z = 0$ …..(i)

$3x + ky - 2z = 0$ ….(ii)

$2x + 4y - 3z = 0$ …..(iii)

On solving (i) and (ii)

$2x - 5z = 0$ …..(iv)

On solving (iii) and (iv)

$4y = -2z$

$$\frac{xz}{y^2} = \frac{\frac{5}{2}z \times z}{\frac{z^2}{4}} = 10$$

37.(1) $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0$

Case-I: $\sqrt{x} \geq 3$

$\Rightarrow \quad 2\left(\sqrt{x} - 3\right) + x - 6\sqrt{x} + 6 = 0 \quad \Rightarrow \quad x - 4\sqrt{x} = 0 \quad \Rightarrow \quad x = 0, 16$

As $x \geq 9 \quad \Rightarrow \quad x = 16$

Case-II: $\sqrt{x} < 3 \quad \Rightarrow \quad -2\sqrt{x} + 6 + x - 6\sqrt{x} + 6 = 0 \quad \Rightarrow \quad x - 8\sqrt{x} + 12 = 0$

$\Rightarrow \quad \left(\sqrt{x} - 6\right)\left(\sqrt{x} - 2\right) = 0 \quad \Rightarrow \quad x = 36, 4$

As, $\sqrt{x} < 3 \quad \Rightarrow \quad x = 4$

$\therefore$ There are exactly two elements in the given set.

38.(4) $8\cos x \cdot \left[\left(\cos^2 \frac{\pi}{6} - \sin^2 x\right) - \frac{1}{2}\right] = 1$

$8\cos x\left(\frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x\right) = 1$

$\frac{8\cos x}{4} \times \left(4\cos^2 x - 1 - 2\right) = 1$

$\cos 3x = 4\cos^3 x - 3\cos x$

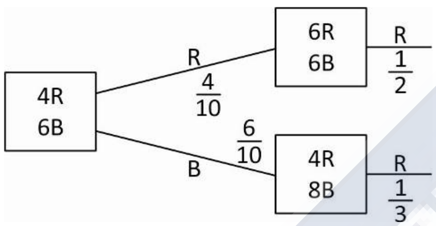
$2 \times \cos 3x = 1$

$\cos 3x = \dfrac{1}{2}$

$3x \in [0, 3\pi]$

$3x = \dfrac{\pi}{3}, 2\pi - \dfrac{\pi}{3}, 2\pi + \dfrac{\pi}{3} \quad \Rightarrow \quad \text{Sum} = \dfrac{13\pi}{9}.$

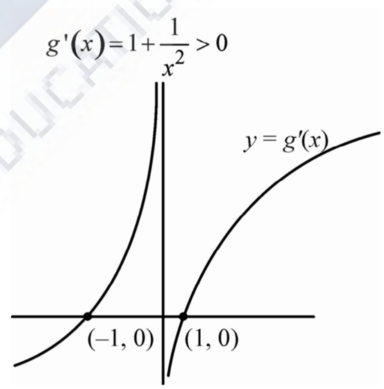
39.(4)



Total probability $= \dfrac{4}{10} \cdot \dfrac{1}{2} + \dfrac{6}{10} \cdot \dfrac{1}{3} = \dfrac{2}{5}$

40.(2) Let $g(x) = x - \dfrac{1}{x} = t$

$g'(x) = 1 + \dfrac{1}{x^2} > 0$



$\therefore \quad t \in R - \{0\}; t^2 \in (0, \infty)$

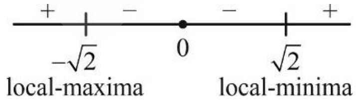
$\therefore \quad f(x) = x^2 + \dfrac{1}{x^2} = \left(x - \dfrac{1}{x}\right)^2 + 2 = t^2 + 2 \in (2, \infty)$

$\therefore \quad h(x) = \dfrac{f(x)}{g(x)}$

$\therefore \quad \dfrac{f(x)}{g(x)} = \dfrac{t^2 + 2}{t} = t + \dfrac{2}{t}$

Let $h(t) = t + \dfrac{2}{t}$

$h'(t) = 1 - \dfrac{2}{t^2}$



$\therefore$ Local minimum value occurs at $t = \sqrt{2}$

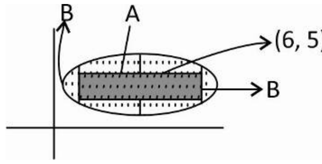
$\therefore$ Local minimum value $= h(\sqrt{2}) = \sqrt{2} + \dfrac{2}{\sqrt{2}} = 2\sqrt{2}$

41.(4) Since Set A is, $|a - 5| < 1 \quad 4 < a < 6$

and $\quad |b-5|<1 \quad 4<b<6$

Now B is

$$\frac{(a-6)^2}{9}+\frac{(b-5)^2}{4}\le 1$$



It can be seen that all vertices of rectangle lie inside the ellipse, therefore $A \subset B$

42.(3) $\sim(p \vee q) \vee (\sim p \wedge q)$

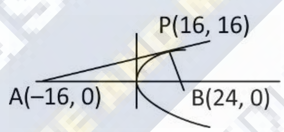
p	q	$\sim(p \vee q)$	$\sim p \wedge q$	$\sim p$
T	F	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	F	T

43.(4) The equation of tangent at P

$$y-16=\frac{1}{2}(x-16) \quad \Rightarrow \quad A \equiv (-16, 0)$$

The normal is $y-16=-2(x-16)$

$B \equiv (24, 0)$



Since $\angle APB = \frac{\pi}{2}$

$\therefore$ AB is the diameter.

Center of the circle $C \equiv (4, 0)$

Slope of $PB = -2 = m_1$

Slope of $CP = \frac{4}{3} = m_2 \Rightarrow \tan\theta = \left|\frac{m_2-m_1}{1+m_2m_1}\right| = 2$

44.(1) $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put x = 0

$$\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3$$

$\boxed{A=-4}$

Put x = 1

$$\begin{vmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = (A+B)(1-A)^2$$

$-3(9-4)-2(-6-4)+2(4+6)$

$-15+20+20=(-4+B)25$

$1=(-4+B)$

$B=5$

45.(2) Let $\sqrt{x^3-1}=y$

$(x+y)^5+(x-y)^5$

$=\left({}^5C_0x^5+{}^5C_1x^4y+.......+{}^5C_5\,y^5\right)+\left({}^5C_0x^5-5C_1x^4\cdot y+........-5C_5y^5\right)$

$=2\left[5C_0x^5+{}^5C_2x^3y^2+{}^5C_4xy^4\right]=2\left[C_0x^5+5C_2x^3y^2+{}^5C_4xy^4\right]$

$=2\left[x^5+10x^3\left(x^3-1\right)+5x\left(x^3-1\right)^2\right]=2\left[x^5+10x^6-10x^3+5x\left(x^6+1-2x^3\right)\right]$

$=2\left[x^5+10x^6-10x^3+5x^7+5x-10x^4\right]=2[1-10+5+5]=2$

46.(1) $a_1+a_5+a_9+........a_{49}=416 \quad \Rightarrow \quad a+24d=32 \quad ......(i)$

$a_9+a_{43}=66 \quad \Rightarrow \quad a+25d=33 \quad ....(ii)$

From (i) and (ii) $d=1$ and $a=8$

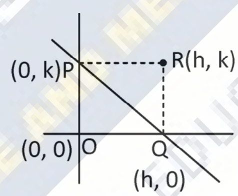
Now, $a_1^2+a_2^2+......a_{17}^2=140m$

$\Rightarrow \quad \sum_{r=1}^{17}\left(8+(r-1)\right)^2=140m \quad \Rightarrow \quad \sum_{r=1}^{17}(7+r)^2=140m \quad \Rightarrow \quad 4760=140m \quad \Rightarrow \quad m=34$

47.(1) Let, $R\equiv(h,k)$

$\therefore \quad P\equiv(0,k)$

$Q\equiv(h,0)$



$\therefore$ Equation of line would be,

$\frac{x}{h}+\frac{y}{k}=1 \quad ...(i)$

$\Rightarrow \quad \frac{2}{h}+\frac{3}{k}=1$

$2k+3h=hk$

Locus of (h,k) is $2y+3x=xy$

48.(2) Given $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$

$f(x)+f(-x)=\frac{\sin^2 x}{1+2^x}+\frac{2^x(\sin^2 x)}{1+2^x}=\sin^2 x=\int_0^{\pi/2}\sin^2 x\,dx=\int_0^{\pi/2}\sin^2 x\,dx=\frac{\pi}{4}$

49.(3) $g(x)=\cos x^2$

$f(x)=\sqrt{x}$

$g(f(x))=\cos x$

Given, $18x^2-9\pi x+\pi^2=0 \quad \Rightarrow \quad (6x-\pi)(3x-\pi)=0$

$\therefore \quad x=\frac{\pi}{6},\frac{\pi}{3}$

Area $=\int_{\pi/6}^{\pi/3}\cos x\,dx=\frac{\sqrt{3}-1}{2}$

50.(1) $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+..........+\left[\frac{15}{x}\right]\right)$

$= \lim_{x\to 0^+} x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}+\frac{2}{x}-\left\{\frac{2}{x}\right\}+.......+\frac{15}{x}-\left\{\frac{15}{x}\right\}\right)$

$= \lim_{x\to 0^+} (1+2+3+........+15) + \lim_{x\to 0^+} x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+........+\left\{\frac{15}{x}\right\}\right)$

Now $0 \le \{x\} < 1 \ \forall\ x \in R = 120$

51.(1) Variance $= \frac{45}{9}-(1)^2 \qquad = 5-1=4$

$\sigma = \sqrt{\text{Variance}} = 2$

52.(4) $\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$

$\int \frac{\tan^2 x\ \sec^6 x}{\left(\tan^5 x+\tan^2 x+\tan^3 x+1\right)^2}dx$

Put $\underline{\tan x = t} \qquad \Rightarrow \qquad \sec^2 x = \frac{dt}{dx}$

$\int \frac{t^2 \quad (1+t^2)^2}{(t^3+1)^2(t^2+1)^2}\,dt$

$t^3+1=y$

$3t^2=\frac{dy}{dt}$

$\frac{1}{3}\int\frac{dy}{y^2}=-\frac{1}{3(y)}+C=-\frac{1}{3(\tan^3 x+1)}+C$

53.(3) Doubtful points for differentiability are 0 and π

At $x=0$

$f'(0^+)= \lim_{h\to o^+}\frac{|h-\pi|\times(e^{|h|}-1)\times\sin|h|-0}{h}$

$= \lim_{h\to o^+}\frac{(\pi-h)\ \times(e^h-1)\times\sin h}{h}$

$\because \lim_{h\to 0^+}\frac{\sinh}{h}=1$ and $\lim_{h\to o^+} e^h-1=0$

$\therefore \qquad f'(0^+)=\pi\times 0\times 1=0$

$f'(0^-)= \lim_{h\to o^+}\frac{|-h-\pi|\times(e^{|-h|}-1)\times\sin|-h|-0}{-h}$

$= \lim_{h\to 0^+}\frac{(\pi+h)\ \times(e^h-1)\times\sin h}{-h}$

$\because \lim_{h\to 0^+}\frac{\sinh}{h}=1$ and $\lim_{h\to o^+} e^h-1=0$

$\therefore \qquad f'(0^-)=(-\pi)\times 0\times 1=0$

$\therefore \quad f'(0^+) = f'(0^-) = 0$

Similarly $f'(\pi^+) = f'(\pi^-) = 0$

Hence $f(x)$ is differentiable $\forall\ x \in R$

54.(1) $\frac{dy}{dx} + y\ cot\ x = 4x\ cosec\ x \quad \Rightarrow \quad d\left(y\ sin\ x\right) = 4xdx$

Integrating both sides we get: $y\ sin\ x = 2x^2 + c$

Also, $y\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad c = -\frac{\pi^2}{2}$

$\Rightarrow \quad y\ sin\ x = 2x^2 - \frac{\pi^2}{2} \quad \Rightarrow \quad y\left(\frac{\pi}{6}\right) = -\frac{8\pi^2}{9}$

55.(3) $\vec{u}\cdot(\vec{a}\times\vec{b}) = 0; \quad \vec{u}\cdot\vec{a} = 0$ and $\vec{u}\cdot\vec{b} = 24.$

Let $\vec{b} = (\vec{b}\cdot\hat{a})\hat{a} + (\vec{b}\cdot\hat{u})\hat{u}$

$|\vec{b}|^2 = (\vec{b}\cdot\hat{a})^2 + (\vec{b}\cdot\hat{u})^2$

$|\vec{b}|^2 = (\vec{b}\cdot\hat{a})^2 + \frac{(\vec{b}\cdot\hat{u})^2}{|\hat{u}|^2}$

$2 = \frac{2}{7} + \frac{(24)^2}{|\hat{u}|^2} \quad \Rightarrow \quad |\vec{u}|^2 = 336$

56.(2) $\frac{x-5}{1} = \frac{y+1}{1} = \frac{z-4}{1} = \lambda$

$P\left(\lambda+5,\ \lambda-1,\ \lambda+4\right)$

P is foot of perpendicular from A to plane $3\lambda + 8 = 7$

$\lambda = -\frac{1}{3}$

$P\left(\frac{14}{3}, \frac{-4}{3}, \frac{11}{3}\right)$

$\frac{x-4}{1} = \frac{y+1}{1} = \frac{z-3}{1}$

$Q\left(\lambda+4, \lambda-1, \lambda+3\right)$

Q is foot of perpendicular from B to plane

$3\lambda + 6 = 7$

$\lambda = \frac{1}{3}$

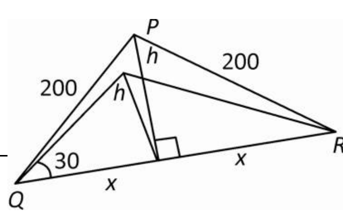
$Q\left(\frac{13}{3}, \frac{-2}{3}, \frac{10}{3}\right)$

$\therefore \quad PQ = \frac{\sqrt{1+4+1}}{3} = \frac{\sqrt{6}}{3} = \sqrt{\frac{2}{3}}$

57.(3) $\frac{h}{x} = \frac{1}{\sqrt{3}}$

$x = \sqrt{3}h$

$200 = 3h^2 + h^2$



$4h^2 = (200)^2$

$4h^2 = 40000$

$h = 100$

58.(3) ${}^6C_4 \cdot {}^3C_1 \times 1 \times 4!$

$\frac{6 \times 5}{2} \cdot 3 \times 24 \equiv 45 \times 24 = \boxed{1080}$

59.(4) $A = 1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + A^2 + 2.20^2$

$= \left(1^2 + 2^2 + 3^2 + 4^2 .... + 20^2\right) + \left(2^2 + 4^2 + ... + 20^2\right)$

$= \frac{20 \times 21 \times 41}{6} + 4 \times \frac{10 \times 11 \times 21}{6} = 2870 + 1540 = 4410 = 2870 + 1540 = 4410$

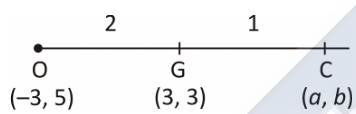
$B = \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6} = 540 \times 41 + 41 \times 280 = 41 \times 820 = 33620$

$33620 - 8820 = 100\lambda$

$100\lambda = 24800$

$\lambda = 248$

60.(1)



$\frac{2a-3}{3} = 3 \quad \Rightarrow \quad 2a = 12 \Rightarrow a = 6$

$\frac{2b+5}{3} = 3 \quad \Rightarrow \quad 2b = 4 \Rightarrow b = 2$

$AC = \sqrt{(6+3)^2 + 3^2}$

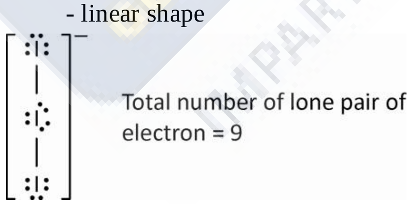
Diameter $= AC = \sqrt{81+9} = \sqrt{90}$

Radius $= \frac{3\sqrt{10}}{2} = \frac{3 \times \sqrt{10}}{\sqrt{2} \times \sqrt{2}} \qquad = 3\sqrt{\frac{5}{2}}$

PART-C	CHEMISTRY

61.(1) I_3^- is - sp^3d hybridised

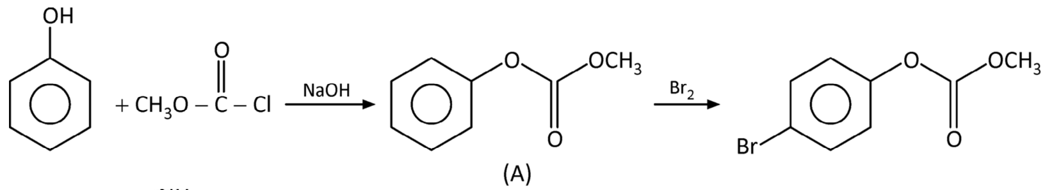
- linear shape

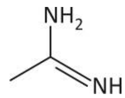


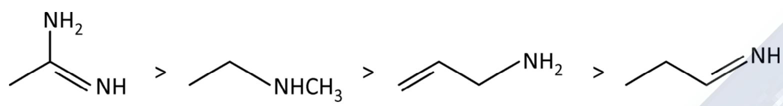
62.(4) CH_3COOK is a salt of a weak acid and a strong base

$\therefore$ Most basic

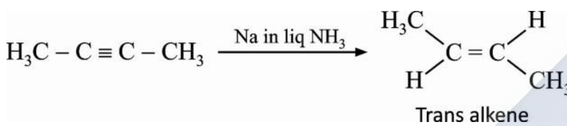
63.(1)



64.(1) Amidines,  are stronger organic bases.



65.(1) Methyl orange is used for titration of strong acid and weak base.

66.(1) 

67.(2) $C_xH_yO_z$ has z oxygen atom

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow x\,CO_2 + \frac{y}{2}H_2O$$

O atoms required for combustion $= 2\left(x+\frac{y}{4}\right)$

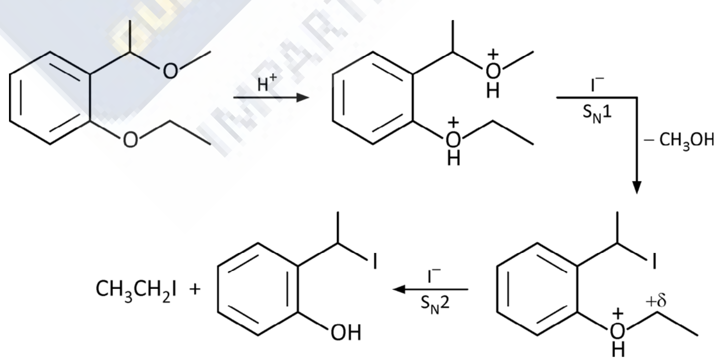
$$z = \frac{1}{2}\left[2\left(x+\frac{y}{4}\right)\right]$$

$$z = x+\frac{y}{4}$$

68.(1) During reduction $H_2O_2 \longrightarrow H_2O$

During oxidation $H_2O_2 \longrightarrow O_2$

69.(2)



Option (2) is correct [NCERT Class XII Part-II, Page No.-340]

70.(1) $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

$nB_2H_6 = \frac{27.66}{27.66} = 1$

n_{O_2} required = 3

$2H_2O \longrightarrow 2H_2 + O_2$

n-factor for O_2 = 4

$\therefore$ Number of equivalent = $3 \times 4 = 12F = 12 \times 96500\,C$

$i \times t = 12 \times 96500$

$t = \frac{12 \times 96500}{100} s = \frac{12 \times 96500}{100 \times 3600} h = 3.2\,hr$

71.(3) $\Delta G = \Delta H - T\Delta S$

$-RT\ell nk = \Delta H - T\Delta S$

$\ell nk = \frac{-\Delta H}{RT} + \frac{\Delta S}{R}$

Slope is $\frac{-\Delta H}{R}$

Since ΔH is $-$ve

$\therefore$ Slope is positive.

72.(3) $r = k[A]^n$

$1 = k(363 \times 0.95)^n$ ....(i)

$0.5 = k(363 \times 0.67)^n$ ....(ii)

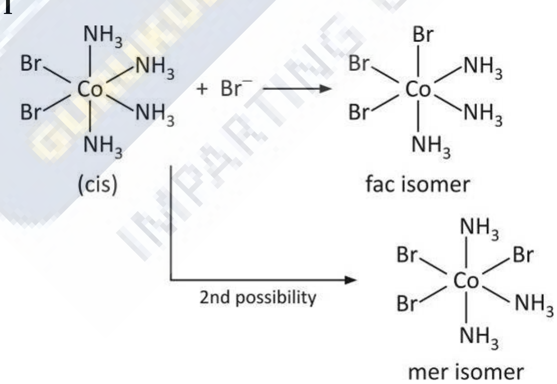
From (i) and (ii)

$n = 2$

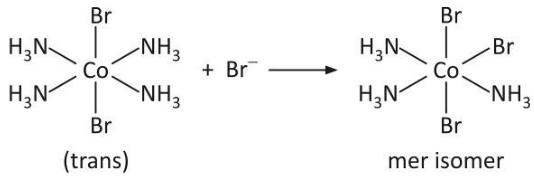
73.(3)

Option (3) is correct [NCERT Class XII Part-II, Page No 405]

74.(4) **Case - I**

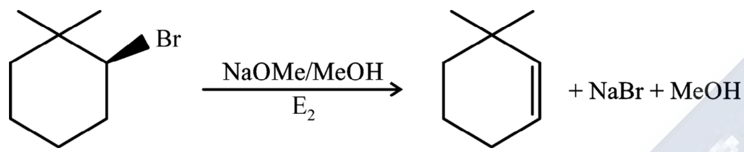


Case - II

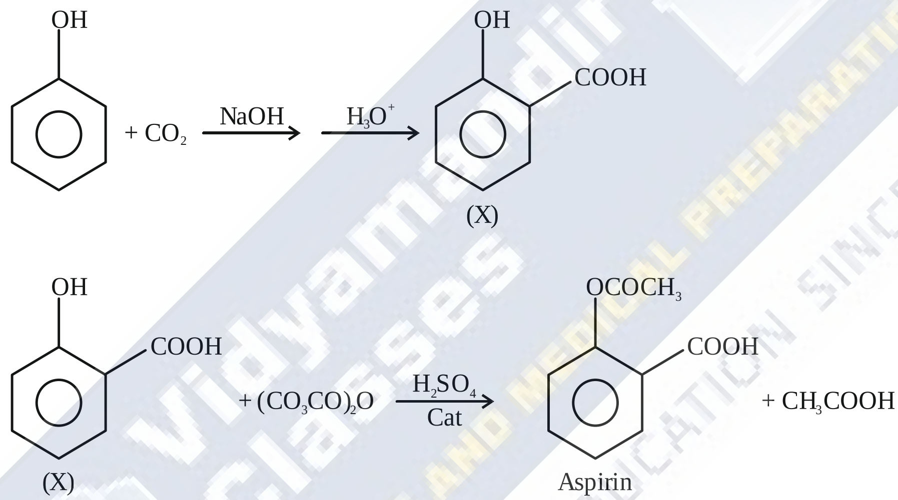


∴ Two isomers (fac and mer) are produced if reactant complex ion is a cis isomer.
Only one isomer (fac) is formed if reactant complex ion is a trans isomer.

75 (4).



76 (3).



[NCERT class XII part II/Page No. 330]

77.(1) $BaSO_4(s) \rightleftharpoons Ba^{2+}(aq) + SO_4^{2-}(aq)$ $\quad K_{sp} = 10^{-10}$

$Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$

Conc. of SO_4^{2-} in final solution $= \dfrac{50 \times 1}{500} = 0.1M$

For final solution

$\Rightarrow \quad [Ba^{2+}][SO_4^{2-}] = 10^{-10} \quad \Rightarrow \quad [Ba^{2+}] = 10^{-9}M$

$M_iV_i = M_fV_f$

$C \times 450 = 10^{-9} \times 500 \Rightarrow C = 1.1 \times 10^{-9}M$

78.(4) and Kjeldahl method is not applicable to compounds containing nitrogen in nitro (NO_2) and azo (N = N –) groups nitrogen present in the ring (pyridine) as nitrogen of these compounds does not change to ammonium sulphate.

[NCERT Class XI part II/Page No. 358]

79.(1) $\underset{(M)}{Al} + NaOH \longrightarrow \underset{\substack{\text{white gel ppt} \\ (X)}}{Al(OH)_3} \xrightarrow{NaOH} NaAlO_2$

$Al(OH)_3 \xrightarrow{\Delta} Al_2O_3$

Al_2O_3 is used in chromatography as an absorbent. (Refer NCERT Class XIth/Part-II, Page-352)

80.(4) $HCl \longrightarrow H^+ + Cl^-$

$\quad\quad\quad\quad 0.2M \quad 0.2\,M$

$H_2S \rightleftharpoons H^+ + HS^- \qquad K_1 = 10^{-7}$

$HS^- \rightleftharpoons H^+ + S^{2-} \qquad K_2 = 1.2 \times 10^{-13}$

$H_2S \rightleftharpoons 2H^+ + S^{2-} \qquad K = K_1 \cdot K_2$

$\qquad = 1.2 \times 10^{-20}$

$$K = \frac{[H^+]^2[S^{2-}]}{[H_2S]} \qquad [H^+] = 0.2M,\ [H_2S] = 0.1$$

$$1.2 \times 10^{-20} = \frac{(0.2)^2[S^{2-}]}{0.1} \quad \Rightarrow \quad [S^{2-}] = 3 \times 10^{-20}\,M$$

81.(1) (Refer NCERT Class XIth Part-II, Page-407)

The F^- ions make the enamel on teeth much harder by converting hydroxyapatite, $[3Ca(PO_4)_2 \cdot Ca(OH)_2]$, into much harder fluorapatite i.e. $[3Ca(PO_4)_2 \cdot CaF_2]$

82.(2) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} NH_3 + H_2SO_4$

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

83.(3) NCERT Class XII/Part-II, Page No. 443

84.(1) $[Cr(H_2O)_6]Cl_3$

$x + 0 - 3 = 0$

$x = +3$

$[Cr(C_6H_6)_2]$

$x + 0 = 0$

$x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2 + x - 2 - 4 - 2 + 0 = 0$

$x = +6$

85.(1) Pressure of cation in interstitial sites is 'Frenkel' defect.

86.(2) $C_6H_6(\ell) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(\ell)$

$$\Delta n_{(g)} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_{(g)}RT$$

$$= -3263.9 - \frac{1.5 \times 8.314 \times 298}{1000} = -3267.6\ kJ/mol$$

87.(2) BCl_3 and $AlCl_3$ are e^- deficient and thus act as Lewis acid

88.(1) KCl exist as K^+ and Cl^-

89.(2) Depression in freezing pt

$\Delta T_f = i K_f m$

Less the value of i,

Higher the value of freezing pt.

For (2) i = 1 (min)

90.(2) H_2^{2-} does not exist as Bond order is zero

Electronic configuration of H_2^{2-} : $\sigma_{1s}^2 \sigma^*{}_{1s}^2$

$B.O = \frac{2-2}{2} = 0$